HUMAN JUDGMENT AND IMPRECISE PROBABILITIES

MICHAEL J. SMITHSON

1. Introduction

The study of human judgment under uncertainty has a history that is almost contemporaneous with that of probability theories. This is not a coincidence. From the outset, the idea of using probability to describe cognitive states or aspects of subjective judgment has provoked debate, theory construction, and empirical research. It is no exaggeration to say that probability theories have exerted a strong prescriptive influence on the study of judgment and decision making (see Gigerenzer 1994 [21] and Smithson 1989 [41] for overviews).

In the modern era, proponents of the Subjective Expected Utility (SEU) framework advocated a version of Bayesianism as the benchmark for rational judgment and decision making, and this viewpoint dominated studies of human judgment and decision making during the 50's and 60's. By the late 70's and early 80's, some scholars had begun to question whether we should regard deviations from probability theories as "irrational" (cf. Cohen 1981 [9], Jungermann 1983 [29]), and attempts to develop descriptive theories of decision making that retained as many features of SEU as possible became a small-scale industry among decision scientists. However, most of these critiques and alternatives have left certain Bayesian prescriptions unquestioned. Two of these are directly relevant to the study of imprecise probabilities:

- Precision, i.e. the doctrine that uncertainty (or utility as well) may be represented by a single number; and
- Prior sample space knowledge, i.e. the assumption that all possible outcomes or alternatives are known beforehand.

Accusations against Bayesians of over-precision and arbitrariness in their priors date back to the mid-19th century, but empirical studies of how people deal with imprecision were rare until the mid-1980's and to this day there are almost no studies of how people cope with sample space ignorance. In terms of reasonable combinations from Table 1, the vast majority of empirical studies deal with situations where probabilities are precise, outcomes are known, and the utilities of all outcomes are precise (cell 1). A much smaller but growing literature concerns situations with vague probabilities but known outcomes (usually with precise utilities—cell 2). A still smaller set of studies deals with imprecise utilities and/or partly known outcomes. Very few studies venture any further into imprecision or ignorance than that (but see, for instance, Hogarth & Kunreuther 1995 [28]).

Early attempts to develop descriptive as well as prescriptive frameworks for decision making when probabilities are unknown (but not necessarily when outcomes

		Probabilities		
Outcomes	**Utilities**	Precise	Vague	Vacuous
Known	Precise	1	2	3
	Vague	4	5	6
	Vacuous	7	8	9
Partly known	Precise	10	11	12
	Vague	13	14	15
	Vacuous	16	17	18
Unknown	Vacuous			19

TABLE 1. Knowledge of Outcomes and Probabilities

also are unknown) include Keynes (1921 [31]) and Knight (1921 [32]). These and other more recent writings have created something of a confusion in concepts and terminology that has yet to be resolved. In most psychological research, writers use "ambiguity" to refer to imprecise probabilities. I will use that term here, although I prefer Max Black's (1937 [3]) use of "vagueness" for this purpose (in the classical tradition philosophers such as Black and literary critics such as Empson (1930 [17]) use "ambiguity" to refer to multiple discrete possible interpretations for a word or object).

Although Knight ([32] p. 219) had posed conundrums for decision makers based on ambiguous probabilities, Ellsberg (1961 [16]) brought this matter to the attention of SEU advocates and the psychological research community. His stated object was to investigate whether the distinction between ambiguous and precise probabilities has any "behavioral significance". His most persuasive example (the one most writers describe in connection with "Ellsberg's Paradox") is as follows. Suppose we have an urn with 90 balls, of which 30 are Red and 60 are either Black or Yellow (but the proportions of each are unknown). If asked to choose between gambles A and B as shown in the upper part of Table 2 (i.e. betting on Red versus betting on Black), most people prefer A.

	30	60	
	Red	Black	Yellow
A	\$100	\$0	\$0
B	\$0	\$100	\$0

	30	60	
	Red	Black	Yellow
C	\$100	\$0	\$100
D	\$0	\$100	\$100

TABLE 2. Ellsberg's Problem

However, when asked to choose between gambles C and D, most people prefer D. People preferring A to B and D to C are violating one of the SEU axioms (often called the "Sure-Thing Principle") because the (C, D) pair simply adds \$100 for drawing a Yellow ball to the (A, B) pair. If we prefer A to B, we should also prefer C to D.

Thus, Ellsberg demonstrated that ambiguity has behavioral consequences that violate the axioms of SEU. An obvious explanation for the A–D preference pattern is that when probabilities are imprecise people adopt a pessimistic stance towards the possible outcomes. The research literature on this phenomenon calls it "ambiguity aversion", and the past 20 years have witnessed a marked growth in the number studies and attempts to incorporate it into modified SEU frameworks. This literature is summarized in the sections on *Ambiguity aversion* and *Empirical studies of ambiguous probability models*. The review paper by Camerer & Weber (1992 [7]) provides an in-depth and thorough survey.

Sample space ignorance, on the other hand, has received only indirect and unsystematic attention. Research during the late 70's indicated that when alternatives are not explicitly represented in a decision tableau, people tend to underestimate the probability of their occurrence. This was termed the "Catch-All Underestimation Bias" (CAUB) by Fischhoff, Slovic, & Lichtenstein (1978 [18]), and it spawned several studies during the 80's and 90's. A recent framework that generalizes this phenomenon along lines that are contrary to Walley's (1991 [45], 1996 [46]) Embedding and Representation Invariance Principles is Support Theory (Tversky & Koehler 1994 [44]). These and related accounts are described in the section on *Research related to sample space ignorance*.

2. Ambiguity aversion

This review is necessarily less thorough-going and briefer than the excellent survey by Camerer & Weber (1992 [7]). It does, however, contain a few updates. As they observe, one kind of empirical work on ambiguous probabilities has consisted of replications of Ellsberg's original thought-experiments. Ambiguity aversion has been found not only under the conditions set up by Ellsberg (cf. Table 3 in Camerer & Weber 1992 for a list of studies) and gambles with real payoffs, but also even after people are exposed to written arguments persuading them not to indulge in it (e.g. MacCrimmon 1968 [34], Slovic & Tversky 1974 [40]). Some studies report less ambiguity aversion for losses than gains (Cohen, Jaffray, & Said 1985 [10] and Einhorn & Hogarth 1986 [15]), but at least one has found no such difference (Kahn & Sarin 1988 [30]).

The major exceptions to ambiguity aversion appear to be ambiguity *preference* at low probabilities under the prospect of gain and high probabilities under the prospect of loss (Curley & Yates 1985 [11], Einhorn & Hogarth 1985 [14], Kahn & Sarin 1988 [30], and Hogarth & Einhorn 1990 [27]). However, as Heath & Tversky (1991 [25]) and Boiney (1993 [4]) have pointed out, these findings may be due to a 'regression' effect, whereby people infer that an ambiguous probability has a skewed distribution, thereby biasing its mean upward from the lower end or downward from the upper end of the $[0, 1]$ interval.

A number of writers have attempted psychological explanations of ambiguity aversion. Many have taken their cue from Ellsberg's claim that this phenomenon is both reasonable and independent of probability judgments per se. Such a claim goes beyond the trite assertion that the imprecision of a probability may vary independently of the magnitude of the probability. If people who prefer to receive $10 for sure to a coin-toss for $20 or nothing, also prefer the coin-toss to betting $20 on drawing a Red ball from an urn containing an unknown number of Red and Black balls, then there is nothing that psychologically distinguishes ambiguity aversion from risk aversion. Several investigators have found only very low correlations between people's risk attitudes and their attitudes towards ambiguity (Cohen, Jaffray, & Said 1985, Hogarth & Einhorn 1990 [27], Schoemaker 1991 [38]), which would seem to indicate that ambiguity aversion is a distinct phenomenon. Moreover, Sherman (1974 [39]) found a modest correlation between ambiguity aversion and a psychometric scale measuring "intolerance" of ambiguity (developed by Budner 1962 [6]).

Nevertheless, some evidence indicates that ambiguity aversion operates somewhat similarly to risk-aversion. First, increased ambiguity may increase perceived riskiness. Second, ambiguity seems to exhibit some framing effects that are analogous to those found in precise probability judgments. Smithson (1989 [41]) reports two relevant experiments: one in which subjects showed a tendency to perceive an ambiguous risk couched in terms of success as less risky than an equivalent risk couched in terms of failure; and another in which they tended to rate a prospect couched in terms of possibility as less restricting than one couched in terms of necessity. Third, several investigators (Casey & Scholz 1991 [8], Gonzalez-Vallejo et al. 1996 [24], Kuhn & Budsecu 1996 [33], and Smithson 1989 [41]) present evidence that people do not find imprecise *outcomes* more or less important than imprecise probabilities, and that they respond similarly to either kind of imprecision.

Explanations for ambiguity aversion have foundered to some degree on the shoals of definitions. Since various writers have used "ambiguity" to mean rather different things, their explanations sometimes talk past one another. Proposals fall mainly into two camps. One emphasizes the idea that people respond to ambiguity on the basis of preferences. The other claims that their responses are due to an impact that ambiguity has on people's perceptions of likelihood and therefore risk. These explanations are, of course, not incompatible.

The preferences camp comprises a number of competing explanations, whose common thread is the supposedly negative consequences that people attribute to ambiguous situations. Frisch & Baron (1988 [20]), for instance, see ambiguity as a matter of missing information "that is relevant and could be known", and therefore think ambiguity aversion arises from generalizing a heuristic to avoid placing bets when one lacks information that others might have. Einhorn & Hogarth (1985 [14]), on the other hand, consider that disagreements or conflicting assessments cause ambiguity by way of a decrement in source credibility. In a somewhat similar vein, Curley, Yates, & Abrams (1986 [12]) argue that self-justification accounts for ambiguity aversion; while Heath & Tversky (1991 [25]) produce evidence suggesting that ambiguity aversion disappears when people believe they have sufficient knowledge or skill in the relevant domain. All of these explanations imply that the decision maker would prefer to obtain more information or choose the option about which they are best informed, all other things being equal (Baron & Frisch 1994 [1] p. 280).

If that is all there is to explaining ambiguity aversion, then the particular form the ambiguity takes or its underlying cause should not matter. But perhaps it does. Studies underway by Smithson and his colleagues (Smithson, in preparation [42]) strongly support the hypotheses that people prefer consensual but ambiguous assessments to disagreeing but precise ones, and that they regard agreeing but ambiguous sources as more credible than disagreeing but precise ones. Thus, people prefer ambiguity to conflict even when the one is informationally equivalent to the other, possibly because they associate consensus among information sources with source knowledgeability and credibility—i.e. *conflict aversion*. Conflict aversion, attributions of competence, and concerns about accountability or justification all point to the importance of taking into account the social context in which risk information is provided.

Despite arguments presented by writers who favor a preference-based account of ambiguity aversion (e.g. Winkler 1991 [47]), researchers have also produced evidence that ambiguity influences perceptions of likelihood or risk. Einhorn & Hogarth (1985 [14]) provide one of the most convincing demonstrations of these effects and propose a model to describe them. Briefly, they claim that when people are given a probability that they believe is ambiguous, they use it as an anchor and then adjust it upwards or downwards depending on whether they are pessimistic or optimistic about the likelihood of the event concerned. Moreover, the net adjustment depends on the magnitude of the anchoring estimate; low probabilities tend to be adjusted upwards and high ones downwards. This model and others are surveyed in the section on Empirical Studies of Ambiguous Probability Models. The main implication arising from such research is that a utility- or preference-based explanation of ambiguity aversion is necessarily incomplete, which then opens up a debate over whether ambiguity aversion is 'rational' or not.

3. Research related to sample space ignorance

As suggested in the Introduction, there has been little research directly on what people do under sample space ignorance. A group of studies and a framework that address partial sample space ignorance to some extent includes research on the "Catch-All Underestimation Bias" (CAUB), first studied by Fischhoff, Slovic, & Lichtenstein (1978 [18]), and a descriptive framework called Support Theory, recently proposed by Tversky & Koehler (1994 [44]).

Fischhoff et al. conducted experiments concerning people's assignments of probabilities to possible causes of a given outcome (e.g. an automobile that will not start), and found that those possible causes that were explicitly listed received higher probabilities than those that were implicitly incorporated into a "Catch-All" category of additional causes. At least three explanations have been proposed for this effect:

1. Unlisted causes are not as available to a person's mental representation of the situation, and therefore not rated as highly likely;
2. People may perceive ambiguity in a list that is incomplete and mentally redefine some of the items on the list by adding other unlisted causes to them (Hirt & Castellan 1988 [26]); and
3. A list that is incomplete may be perceived as lacking credibility, so people inflate the probabilities of the explicitly listed causes (Dube-Rioux & Russo 1988 [13]).

Russo & Kozlow (1994 [37]) conducted further studies and found the most evidence for the unavailability explanation and the least for the credibility explanation. However, Bonini & Caverni (1995 [5]) provided evidence from the literature and their own experiments that casts doubt on all three explanations. For instance, they found that making the unlisted causes more available to people did not decrease the CAUB.

In a similar vein, Support Theory (Tversky & Koehler 1994 [44] and Rottenstreich & Tversky 1997 [36]) is a framework that begins with the claim that people do not follow the extensional logic of conventional probability theory. Instead, unpacking a compound event into disjoint components tends to increase the perceived likelihood of that event. An immediate implication is that unpacking an hypothesis and/or repacking its complement will increase the judged likelihood of that hypothesis. Moreover, while the sum of the subjective probabilities of an hypothesis and its complement might sum to 1, finer-grained partitions of either will result in 'probabilities' whose sum exceeds 1. Support Theory revives the Keynesian (1921 [31]) distinction between the balance of evidence in favoring a given proposition and the weight or strength of evidence for that proposition.

Both the CAUB and Support Theory are important because they suggest that people are sensitive to the sample space in which events are embedded. If true, then people may violate both the Embedding and Representation Invariance Principles. Moreover, it is possible that they could exhibit a form of "ignorance aversion" akin to ambiguity aversion.

Clearly what is needed here are direct tests involving sample space ignorance rather than merely ambiguous probabilities in situations where the possible outcomes are fully specified. Smithson et al. (in preparation [43]) are currently conducting studies to perform such tests. Preliminary findings point toward the following assertions:

- Most people exhibit "ignorance aversion" when choosing between bets involving only partial ignorance with vacuous probabilities and bets involving sample space ignorance. That is, they prefer partial ignorance when betting on an event that has been named as a possible outcome, and prefer total ignorance when betting on an event other than one that has been named as a possible outcome.
- However, many people endorse Walley's Vacuous Priors Principle (i.e. they give a lower probability of 0 and an upper probability of 1 to any event in the absence of any prior information).
- Likewise, many people give 0 as a lower probability for an unobserved event after having seen other events occur. Conversely, almost no one gives a lower probability of 0 for an event that has already been observed.
- Nevertheless, they do not adhere to the Representation Invariance Principle in their upper probability assignments, giving greater upper probabilities to "any new" event than to a specific named event.
- Most people also violate the Representation Invariance Principle by rating more 'plausible' unobserved events as more likely to occur than less 'plausible' ones.
- The nearest result to a CAUB effect is a tendency for the difference between people's upper and lower probabilities for an unobserved event to be less than that for an observed event. In other words, people tend to be less imprecise about unobserved event probabilities, which suggests that they may underestimate their likelihood.

4. Empirical studies of ambiguous probability models

Camerer & Weber (1992 [7]) provide a thorough review of ambiguous probability models, so this section merely summarizes their review and adds a few remarks pertaining to imprecise probabilities and sample space ignorance. Camerer & Weber group these models three classes:

- Models assuming a single second-order probability (SOP) distribution, which effectively treat possible probabilities as possible outcomes are treated in SEU;
- Models assuming sets of probabilities but not a unique SOP over these sets, which then model preferences in terms of considerations based on all or some of the possible probability distributions in the set; and
- Models based on nonlinear weighting functions of unique probabilities or non-additive probabilities, in which the weighting function expresses ambiguity aversion or preference.

The SOP models (e.g. Kahn & Sarin 1988 [30], Becker & Sarin 1990 [2]) usually assume a well-specified second-order distribution but permit nonlinear weighting functions and relax a few other SEU assumptions. One of the most interesting examples of this approach is one based on Rank-Dependent Expected Utility theory (RDEU: cf. Quiggin 1993 [35]). Unlike earlier probability weighting schemes, this one works with a (de)cumulative distribution function (CDF) whose ordering of outcomes is determined by the decision maker's preferential rank-ordering of them. Perhaps the most interesting prediction by RDEU is that people overweight extremely good and bad events (i.e. the tails of the CDF), which contrasts with models such as Einhorn & Hogarth's (1985 [14]), whose weightings are a function of the magnitude of the anchoring probability rather than the utility of the outcome. An intriguing corollary is that under sample space ignorance people may give higher upper probabilities to an extremal unobserved event than a nonextremal one. Studies are currently underway to investigate this prediction.

Models based on sets of probabilities date back to Ellsberg (1961 [16]) and researchers in this group tend to focus on whether people use some weighted averaging or minimax type of rule (e.g. Gilboa & Schmeidler 1989 [23]) for assessing the utility of each alternative. These models are the most obviously compatible, in formal terms, with imprecise probability theory. A common objection by researchers to both this and the SOP approach is that they replace an unrealistic precision about probability with unrealistic precision about probability bounds. However, as Camerer & Weber ([7] p. 346) suggest, people may not be uncomfortable giving precise bounds; and current studies by Smithson et al. (in preparation [43]) indicate that people are most comfortable and best calibrated when giving lower and upper bounds and a 'best guess'.

The third group of models, namely those using nonadditive probabilities, includes the most popular attempts to account for the apparent effects of ambiguities on risk perception. Many of these are formal models that are compatible to with imprecise probability theory because they employ special cases of imprecise probabilities (e.g. Gilboa 1989 [22]). Others are more descriptively oriented models that have undergone some empirical tests. A simple example is Einhorn and Hogarth's (1985 [14]) model, for which they begin with an anchoring probability P, and then introduce two parameters, θ and β, in the following roles:

$$k_g = \theta(1 - P)$$

and

$$k_s = \theta P^{\beta},$$

where $0 \leq \theta \leq 1$ and $\beta > 0$. According to the EH model, people adjust P upwards by k_g and downward by k_s. So, a person's subjective probability P_s is

$$P_s = P + k_g - k_s = P + \theta(1 - P - P^{\beta}).$$

Einhorn and Hogarth then estimate θ and β from sample judgments of P_s, when P is known.

One may recast the EH model in terms of upper and lower probabilities for this model if P is known and there are sufficient data to estimate θ and β. Let P_- and P^+ represent lower and upper probabilities, respectively. Put

$$P_- = P - \theta P,$$
$$P^+ = P + \theta(1 - P),$$

and

$$P_s = \lambda P_- + (1 - \lambda)P^+.$$

If we set $\lambda = P^{\beta}$, then we have the EH model. Of course, λ could have other functional forms.

The θ parameter is the degree of ambiguity or latitude around P attributed by the person. β, on the other hand, reflects the relative weighting of values smaller than or larger than P. So it may be interpreted as a pessimism/optimism parameter. If P is the chance of getting a reward, then $\beta \leq 1$ would indicate pessimism, and $\beta > 1$ optimism. Fobian & Christensen-Szalanski (1994 [19]) applied the EH model to studying the effects of ambiguity on the likelihood of a negotiated settlement to a liability dispute. They found that reducing ambiguity actually decreased the likelihood of a negotiated settlement, and that which party had greater ambiguity affected the likelihood of settlement conditional on the perceived probability of a victory by the plaintiff.

Walley's (1996 [46]) Imprecise Dirichelet (ID) model may serve as a normative benchmark in research of this kind, and it provides a good case in point both because of its simplicity and ease of comparability with human performance. There is a direct connection between the EH and ID models. Given N observations of which n are occurrences of the outcome concerned, if we set $\theta = s/(N + s)$ for $s > 0$ and estimate P by putting $P = n/N$, then P_s is a convex combination of Walley's lower and upper probabilities. So under these conditions, a sufficient sample of judgments of P_s enables estimates of s. Figure 1 shows Walleyan lower and upper probabilities (for $N = 7$, $s = 3$) in a graph with 'optimistic' ($\beta = 4$) and 'pessimistic' ($\beta = 0.25$) EH models.

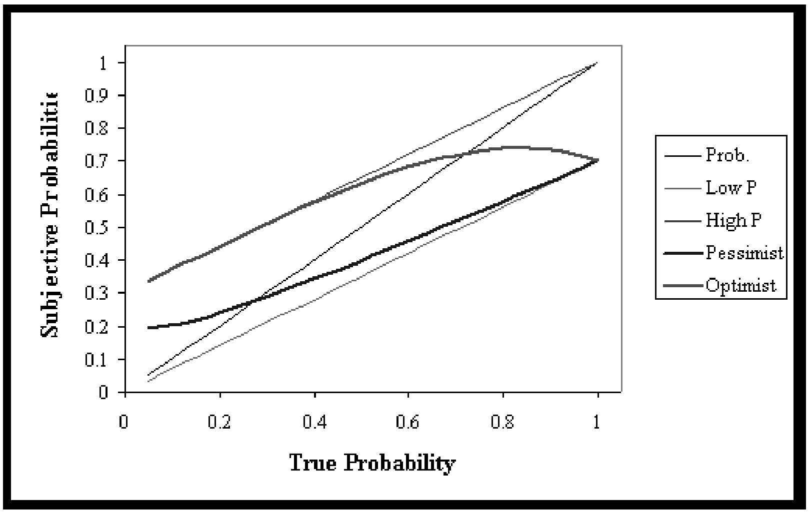

FIGURE 1. EH Models with ID benchmarks

However, researchers are better off obtaining subjective judgments of P_- and P^+ under the conditions specified above. We may obtain direct estimates of s, and ascertain whether people update P_- and P^+ as if they have constant s as N increases (preliminary evidence indicates that many people may update overly cautiously, so that s increases with N). We may also assess whether people are optimistic or pessimistic, independent of their s-values. Setting $P = n/N$, since $P_- = P-\theta P$ and $P^+ = P+\theta(1-P)$, we have $P_-/(1-P^+) = P/(1-P) = n/(N-n)$, so we may use the odds-like ratio $P_-/(1-P^+)$ to assess calibration. Values greater than $P/(1-P)$ indicates optimism and lower ones indicate pessimism. Finally, if we obtain not only P_- and P^+ but also 'best guess' judgments, then we may model P_s as a weighted combination of P_- and P^+ in the setting of the EH model or, conversely, P_- and P^+ as a function of P_s and s.

Despite the obvious interest from psychology and related disciplines in empirical investigations and theories of imprecise probabilities, both research and theory to date have been limited in crucial respects:

- Almost no researchers have elicited lower and upper probabilities from subjects, nor entertained the possibility that these might be differently affected by factors influencing ambiguity aversion;

- Few researchers have distinguished mere ambiguity from ignorance of possible outcomes, so sample space ignorance effects remain largely unexplored;
- Very little attention has been paid to the issue of how people (ought to) update ambiguous probabilities on the basis of new sample information; and
- SEU and precise probabilities are still taken to be the benchmarks of rationality by many empirical researchers.

Studies are underway to investigate the issues raised here, and there is clearly considerable potential for productive dialog between proponents of normative and/or descriptive frameworks, and empirical researchers.

References

[1] J. Baron and D. Frisch. Ambiguous probabilities and the paradoxes of expected utility. In G. Wright and P. Ayton, editors, *Subjective Probability*, pages 273–294. Wiley, Chicester, U.K., 1994.

[2] J. L. Becker and R. K. Sarin. Economics of ambiguity in probability. May 1990. Working Paper, UCLA Graduate School of Management.

[3] M. Black. Vagueness: an exercise in logical analysis. *Philosophy of Science*, 4:427–455, 1937.

[4] L. G. Boiney. Effects of skewed probability of decision making under ambiguity. *Organizational Behavior and Human Decision Processes*, 56:134–148, 1993.

[5] N. Bonini and J.-P. Caverni. The "catch-all underestimation bias": Availability hypothesis vs. category redefinition hypothesis. *Current Psychology of Cognition*, 14:301–322, 1995.

[6] S. Budner. Intolerance of ambiguity as a personality variable. *Journal of Personality*, 30:29–50, 1962.

[7] C. Camerer and M. Weber. Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of Risk and Uncertainty*, 5:325–370, 1992.

[8] J. T. Casey and J. T. Scholz. Boundary effects of vague risk information on taxpayer decisions. *Organizational Behavior and Human Decision Processes*, 50:360–394, 1991.

[9] L. J. Cohen. Can human irrationality be experimentally demonstrated? *Behavioral and Brain Sciences*, 4:317–331, 1981.

[10] M. Cohen, J. Jaffray, and T. Said. Individual behavior under risk and under uncertainty: An experimental study. *Theory and Decision*, 18:203–228, 1985.

[11] S. P. Curley and F. J. Yates. The center and range of the probability interval as factors affecting ambiguity preferences. *Organizational Behavior and Human Decision Processes*, 36:272–287, 1985.

[12] S. P. Curley, F. J. Yates, and R. A. Abrams. Psychological sources of ambiguity avoidance. *Organizational Behavior and Human Decision Processes*, 38:230–256, 1986.

[13] L. Dube-Rioux and J. E. Russo. An availability bias in professional judgment. *Journal of Behavioural Decision Making*, 1:223–237, 1988.

[14] H. J. Einhorn and R. M. Hogarth. Ambiguity and uncertainty in probabilistic inference. *Psychological Review*, 92:433–461, 1985.

[15] H. J. Einhorn and R. M. Hogarth. Decision making under ambiguity. *Journal of Business*, 59:S225–S250, 1986.

[16] D. Ellsberg. Risk, ambiguity, and the Savage axioms. *Quartely Journal of Economics*, 75:643–669, 1961.

[17] W. Empson. *Seven Types of Ambiguity*. Penguin, London, 1930. reprinted in 1995.

[18] B. Fischhoff, P. Slovic, and S. Lichtenstein. Fault trees: Sensibility of estimated failure probabilities to problem representation. *Journal of Experimental Psychology: Human Perception and Performance*, 4:330–344, 1978.

[19] C. S. Fobian and J. J. J. Christensen-Szalanski. Settling liability disputes: the effects of asymmetric levels of ambiguity on negotiations. *Organizational Behavior and Human Decision Processes*, 60:108–138, 1994.

[20] D. Frisch and J. Baron. Ambiguity and rationality. *Journal of Behavioral Decision Making*, 1:149–157, 1988.

[21] G. Gigerenzer. Why the distinction between single-event probabilities and frequencies is important for psychology (and vice versa). In G. Wright and P. Ayton, editors, *Subjective Probability*, pages 129–161. Wiley, Chichester, U.K., 1994.

[22] I. Gilboa. Duality in non-additive expected utility theory. *Annals of Operations Research*, 19:405–414, 1989.

[23] I. Gilboa and D. Schmeidler. Maxmin expected utility with a non-unique prior. *Journal of Mathematical Economics*, 18:141–153, 1989.

[24] C. Goncalez-Vallejo, A. Bonazzi, and A. J. Shapiro. Effects of vague probabilities and of vague payoffs on preference: A model comparison analysis. *JOunral of Mathematical Psychology*, 40:130–140, 1996.
[25] C. Heath and A. Tversky. Preference and belief: Ambiguity and competence in choice under uncertainty. *Journal of Risk and Uncertainty*, 4:5–28, 1991.
[26] E. R. Hirt and N. J. Castellan Jr. Probability and category redefinition in the fault tree paradigm. *Journal of Experimental Psychology: Human Perception and Performance*, 20:17–32, 1988.
[27] R. M. Hogarth and H. J. Einhorn. Venture theory: A model of decision weights. *Management Science*, 36:780–803, 1990.
[28] R. M. Hogarth and H. Kunreuther. Decision making under ignorance: arguing with yourself. *Journal of Risk and Uncertainty*, 10:15–26, 1995.
[29] H. Jungermann. The two camps on rationality. In R. W. Scholz, editor, *Decision Making Under Uncertainty.* Elsevier, Amsterdam, 1983.
[30] B. E. Kahn and R. K. Sarin. Modelling ambiguity in decisions under uncertainty. *Journal of Consumer Research*, 15:265–272, 1988.
[31] J. M. Keynes. *A Treatise on Probability.* Macmillan, London, 1921.
[32] F. H. Knight. *Risk, Uncertainty and Profit.* Houghton Mufflin, Boston, 1921.
[33] K. M. Kuhn and D. V. Budescu. The relative importance of probabilities, outcomes, and vagueness in hazard risk decisions. *Organizational Behavior and Human Decision Processes*, 68:301–317, 1996.
[34] K. R. MacCrimmon. Descriptive and normative implications of the decision-theory postulates. In K. Borch and J. Mossin, editors, *Risk and Uncertainty.* MacMillan, London, 1968.
[35] J. Quiggin. *Generalized Expected Utility: The Rank-Dependent Model.* Kluwer, Dordrecht, 1993.
[36] Y. Rottenstreich and A. Tversky. Unpacking, repacking, and anchoring: Advances in support theory. *Psychological Review*, 104:406–415, 1997.
[37] J. E. Russo and K. Kozlow. Where is the fault in fauilt trees? *Journal of Experimental Psychology: Human Perception and Performance*, 20:17–32, 1994.
[38] P. J. H. Schoemaker. Choices involving uncertain probabilities: tests of generalized utility models. *Journal of Economic Behavior and Organization*, 16:295–317, 1991.
[39] R. Sherman. The psychological difference between ambiguity and risk. *Quarterly Journal of Economics*, 88:166–169, 1974.
[40] P. Slovic and A. Tversky. Who accepts Savage's axiom? *Behavioral Science*, 19:368–373, 1974.
[41] M. Smithson. *Ignorance and Uncertainty: Emerging Paradigms.* Springer, New York, 1989.
[42] M. J. Smithson. Conflict aversion. Working paper, Division of Psychology, The Australian University, in preparation.
[43] M. J. Smithson, K. Takemura, and T. Bartos. Judgment under outcome ignorance. Working paper, Division of Psychology, The Australian University, in preparation.
[44] A. Tversky and D. J. Koehler. Support theory: A nonextensional representation of subjective probability. *Psychological Review*, 101:547–567, 1994.
[45] P. Walley. *Statistical Reasoning with Imprecise Probabilities.* Chapman and Hall, London, 1991.
[46] P. Walley. Inferences from multinomial data: Learning about a bag of marbles. *J. R. Statist. Soc., Series B*, 58:3–57, 1996. with discussion.
[47] R. L. Winkler. Ambiguity, probability, preference, and decision analysis. *Journal of Risk and Uncertainty*, 4:285–297, 1991.