The SIPTA Newsletter

Society for Imprecise Probability:
Theories and Applications

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Message from the editor

This issue of the SIPTA newsletter brings you several treats, ranging from the conclusion of the interview with Prof. Isaac Levi started in the previous issue, to several announcements for events, papers, and tools. However, this issue is saddened by the loss of two characters who have taken part in the community's development from its beginnings.

Henry Kyburg Jr. died on October 20 2007, at the age of 79. The professor of philosophy and computer science was internationally known for important contributions in many different topics, several of which were related to uncertain reasoning. Professor Kyburg was a special figure for this newsletter, as he was the author of the History Section in our very first issue. A short obituary has been posted by the University of Rochester (www.rochester.edu/news/show.php?id=3055), and a review of his contributions, with emphasis on artificial intelligence, appeared in the March 2008 issue of the *AI Magazine*. Longer reviews of his work are planned for a forthcoming special issue of *Synthese*.

We must also note with regret the death of Pauline Coolen-Schrijner on April 23 2008, at the age of 40. Pauline published in a variety of subjects, and produced important results on interval-valued transition matrices and on nonparametric predictive inference. Thomas Augustin was kind enough to prepare a tribute to Pauline, to be found right after this message.

This issue then brings the final part of the interview with Prof. Isaac Levi, on his journey through the land of indeterminate probabilities, and on open problems and challenges. The interview is followed by announcements for ISIPTA'09 and other events and papers, including the recent addition of an electronic version of Kuznetsov's book to the SIPTA web site, and a brief report on the 2008 SIPTA School. Many thanks to Gang Xiang for contributing with a summary of his PhD thesis on algorithms for interval uncertainty; thanks also to Gang's former advisor, Vladik Kreinovich, for suggesting this piece. Finally, there is a Software Section on the DecideIt tool. Quite a substantial issue indeed, thanks to the many contributors.

As always, if you know of any event or publication that should be of interest to members of the society, please let me know (send a message to fgcozman@usp.br).

Cheers!

Fabio G. Cozman

Tribute to Pauline Coolen-Schrijner

by Thomas Augustin

Pauline was born in Arnhem, The Netherlands. She received a M.Sc. degree in Econometrics (main subjects: stochastic processes, operations research and statistics) from the Catholic University of Brabant, and then joined the University of Twente as a Ph.D. student. Her thesis, entitled "Quasi-stationarity of Discrete-Time Markov Chains," was the start of an intensive

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and enduring cooperation with her supervisor, and good friend, Erik A. van Doorn.

In 1995 Pauline continued her career in England, where she worked briefly as Lecturer at Newcastle University and then as Senior Research Associate, Lecturer and Reader in the Department of Mathematical Sciences at Durham University. Her well-recognized research includes contributions to the theory and application of Markov chains, where she worked in particular on quasi-stationary and limiting conditional distributions, and recently also obtained results based on interval-valued transition matrices. In addition, in recent years she was also intensively engaged, together with her husband and colleague Frank Coolen, in the development of nonparametric predictive inference (NPI) — an exciting methodology for predictive inference under low structure assumptions leading to interval-valued probabilities. Paulines favorite areas of application were operations research and reliability, and so one of her main focuses in NPI, also supported by an EPSRC grant, was the development of adaptive replacement and maintenance strategies.

Pauline published over 40 papers in a wide range of journals, including Journal of Applied Probability, Stochastic Models, Journal of Statistical Planning and Inference, Journal of the Operational Research Society, European Journal of Operational Research, Reliability Engineering and System Safety, Journal of Statistical Theory and Practice, and the Journal of Risk and Reliability, as well as invited contributions to the Wiley Encyclopedia of Quantitative Risk Analysis and Assessment, and the Wiley Encyclopedia of Statistics in Quality and Reliability. She served as an Editorial Board member of the Journal of Risk and Reliability from its inauguration in 2006, and co-edited a special issue of the Journal of Statistical Theory and Practice on "Imprecision in Statistical Theory and Practice." She greatly enjoyed working with postgraduate students, and supervised four Ph.D. students: Maha Rahrouh, who completed in 2005, Richard Crossman and Tahani Maturi, who are due to complete in 2009, and Rebecca Baker. Research results which Pauline achieved with her recent Ph.D. students, as well as further results with Frank Coolen, will lead to a substantial number of further papers to be published in the near future.

Far beyond her scientific achievements, we will keep Pauline in our mind, and our heart, as a wonderful, simply loveable person, with a bright and open mind, and a heart of gold. Although in the last two decades her devastating disease(s) had forced Pauline to give up gradually many of the things most precious to her, she was always full of optimism, mental strength and energy, with a special sense for the "small things in life," being well aware that — quoting the last line of her web page — "breathing is not something we can take for granted."

History section: An Interview with Isaac Levi Part II

This is the final part of an email interview with Prof. Levi, centered on his contributions to imprecise and indeterminate probabilities. We thank Prof. Levi for his willingness to contribute to the newsletter.

Fabio G. Cozman

How receptive was the philosophical/statistical community to these ideas, when you started advocating them?

There has been a tendency in the philosophical community to ignore the fairly specific understanding of indeterminacy that I have been proposing and to assimilate the views I favor to the fairly widespread use of sets of probability distributions for other purposes. There are several such uses to consider:

1. Students of robust statistics who are not interested in subjective probability have, nonetheless, used sets of probability functions to represent sets of statistical distributions. I think such representations are important and useful but they should not be confused with sets of personal, subjective or credal probability distributions. Such sets, for example, need not be convex as I (but not the CMU trio) require.

2. Even strict Bayesians who insist that rationality requires that credal probability judgment be numerically determinate acknowledge that our capacity to satisfy this requirement is limited and that deliberating agents may not be able to discern their own credal state or the credal state of others with perfect precision. When seeking to ascertain the extent of the imprecision

in probability judgment, sets of distributions may once more be useful. Such representations of imprecise credal probabilities should not be confused with representations of indeterminate probability. These sets need not be convex.

3. R.C. Jeffrey (1965) proposed an account of preference among propositions that required that the preferences yield a weak ordering. He then sought a representation of these desirabilities in terms of probability-desirability pairs where the same ordering can be represented by distinct probability-desirability pairs. Sometimes the sets of probability components of such pairs are studied and these sets seemed to have been conflated with indeterminate probabilities.

In addition to the diversity of uses to which sets of probability distributions have been put in the past few decades, there have been many authors who deviate from the requirement that probability be determinate by appealing to various versions of intervalism concerning probability. Much thinking of this sort is tied to the promotion of decision theories that argue for maximizing lower expectations and cognate notions all of which presuppose maximality rather than E-admissibility as necessary for choice.

You have pioneered work on E-admissibility and on convexity; how did you conceive these two important (and controversial?) ideas, and how did they become central pieces of your thinking?

My concern with both ideas has always been part of a project to extend the notion of doubt to probability and utility judgment from the clear cases where doubt concerns lack of certainty or full belief. As I have already indicated, I also think that the serious defect in the "Bayesian" outlook that rests on updating according to conditionalization via Bayes' theorem and maximizing expected utility has been the insistence on the strict requirement of confirmational uniqueness. My own efforts have been to explore the development of an approach covering the same bases as the Bayesians covered and using principles that they should be prepared to accept except for confirmational uniqueness and the corresponding uniqueness requirement for extended value structures. Thus, I continue to endorse confirmational conditionalization and E-admissibility as a necessary condition for admissibility. I see the value structure $V(A)$ as derived from the credal state and extended value structure with the aid of an expected utility principle and cross product rule. All of this has a Bayesian flavor. What is missing is the devotion to uniqueness of the expected utility function, the utility function and credal probability function. In this respect, I am rabidly anti Bayesian.

Could you mention a few open problems and challenges that should receive attention from the community?

I became engaged with indeterminacy in probability judgment because of my interest in expansion including inductive expansion of states of full belief. As I mentioned before, this has led me to wonder about the conditions under which confirmational commitments can be legitimately modified. There has been relatively little work on this topic. Yet it seems to me to be of central importance to the foundations of statistical reasoning.

Another issue of even greater difficulty concerns the modification of value commitments. John Dewey maintained that there ought to be inquiry into values just as there is scientific inquiry. But how to approach this problem remains opaque to me.

And, although in my heart of hearts, I am convinced that confirmational convexity ought to hold and that maximality is unacceptable, it is perfectly clear that serious authors like Seidenfeld, Kadane and Schervish who share my commitment to E-admissibility remain unconvinced by the arguments I can muster.

The basis for the disagreement between the CMU trio and myself concerns whether the concept of consensus or suspense we require should be grounded in consensus in the state of full belief, credal state and extended value structure as separable and independently variable components of the decision maker's point of view or whether we should focus directly on consensus in the value structure.

References

References cited in the first part of the interview appeared in the previous issue of the newsletter.

Jeffrey, R.C. (1965), *The Logic of Decision*, New York: Wiley.

The Sixth International Symposium on Imprecise Probability: Theories and Applications ISIPTA '09

The Sixth ISIPTA will happen in 2009, following a series started in 1999. Detailed information about the symposium can be found at the site www.sipta.org/isipta09. The remainder of this section presents a few important itens regarding the meeting.

The ISIPTA meetings are one of the primary international forums to present and discuss new results on the theory and applications of imprecise probabilities. The symposium deals with many mathematical and statistical models and methods, allowing us to measure chance or uncertainty without the restriction of sharp probabilities. These models include lower and upper expectations or previsions, interval-valued probabilities, sets of probability measures, belief functions, Choquet capacities, comparative probability orderings, fuzzy measures, possibility measures, plausibility measures, and sets of desirable gambles. Imprecise probability models are needed in inference and decision problems where the relevant information is scarce, vague or conflicting, and where preferences may be incomplete.

Location and Dates

ISIPTA '09 will be held at Durham University, Collingwood College, in Durham, United Kingdom. Collingwood College provides onsite ensuite accommodation. More information about Collingwood College can be found at www.dur.ac.uk/collingwood/.

Important dates:

- For papers:
  Paper submission: January 30 2009
  Notification of acceptance: March 15 2009
  Revised papers: April 15 2009
- For posters without paper:
  Abstract submission: April 15 2009
  Notification of acceptance: May 1 2009
- Symposium: July 14-18 2009

Format and Submissions

Each accepted paper is to be presented both (i) in a plenary session, where we ask for a short introduction and sketch of the context and relevance of the paper; and (ii) in a poster session, where ample opportunity and time is given for detailed explanation and discussion. There will be no parallel sections.

It is also possible to present a poster without a paper, for those who wish to present and discuss their work within the framework of the conference but whose results are not yet finalized. If you wish to present a poster without paper, you are invited to submit a one-page abstract of the work you intend to present. These abstracts will be made available at the conference and online.

In memory of Henry Kyburg Jr. and Pauline Coolen-Schrijner, two special sessions will be organized. The papers for these sessions will be selected by the steering committee.

Papers can be submitted electronically using the conference website.

Steering Committee

Thomas Augustin (Ludwig-Maximilians University, Germany)
Frank Coolen (Durham University, UK)
Gert de Cooman (Universiteit Gent, Belgium)
Serafin Moral (Universidad de Granada, Spain)
Teddy Seidenfeld (Carnegie Mellon University, USA)
Matthias Troffaes (Durham University, UK)

Program Committee

Thomas Augustin (Ludwig-Maximilians University, Germany)
Frank Coolen (Durham University, UK)
Serafin Moral (Universidad de Granada, Spain)
Matthias Troffaes (Durham University, UK)

Program committee members are listed at the conference site.

Questions and Secretariat

If you have questions about the symposium, please contact (preferably by email):

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Kuznetzov's "Interval Statistical Models" at sipta.org

Vladimir Petrovich Kuznetsov published, in 1991, a 344-page book titled *Interval Statistical Methods*, describing his theory of interval-valued probabilities and expectations. The few researchers who were fortunate to interact with

Kuznetsov have reported on his creativity, and the Russian-speaking fellows in the community have long praised his book. There seem to be many new concepts proposed in the book, and novel mathematical results about them. For instance, we know through sparse translations that Kuznetsov played with several concepts of independence, at least one of which was proposed first by him.

Unfortunately, Kuznetsov died before his ideas became widely known, and before his book could be translated. The translation of his book has not materialized yet.

Recently, SIPTA has received the permission to make Kuznetzov's book freely available This is a very happy event indeed. Digitized versions of the book (in pdf format) are available at SIPTA's web site; more precisely, at www.sipta.org/index.php?id=res#kuz.

The Society is grateful to Igor Petrovich Kuznetzov and Evgeni Vladimirovich Kuznetzov for giving their authorization as legal owners of the copyright of the book, and to Igor Kozine and Enrique Miranda for their help during this process.

The next challenge is to translate the book into other languages. If you can read Russian, you might consider getting yourself involved in a collaborative translation of the book; the wiki of the project can be found at www.sipta.org/kuz/wiki/.

Please contribute to this translation if you can!

Meetings on Imprecise Probabilities

Several interesting meetings have taken place since the publication of the last issue of the newsletter. Some of these meetings have already been alluded to in previous issues, but others deserve to be mentioned here.

The following announcements appeared in the SIPTA mailing list. Note that the Second International Conference on Finite Element Methods in Engineering and Science, which took place in El Paso, Texas (December 8-12, 2008), also welcomed papers on interval and probabilistic uncertainty, even though it did not refer to these terms quite explicitly (thanks to Vladik Kreinovich for pointing this out).

Tutorial on Probabilistic Risk Analysis With Hardly Any Data (2007, 2008)

These tutorials, held in conjunction with the Society for Risk Analysis Annual Meeting (December 9 2007 and December 7 2008), introduced and compared methods for developing a fully probabilistic risk analysis when little or no empirical data are available to inform the risk model. The workshops were organized around the basic problems that risk analysts face: not knowing the input distributions, not knowing their correlations, not being sure about the model itself, or even which variables should be considered. Imprecise probability methods as well as competing strategies were discussed. Further information is available at http://www.ramas.com/nodata.htm, including discussion of interesting work by the presenters, Scott Ferson and W. Troy Tucker.

Third Workshop on Reliable Engineering Computing (REC08)

This workshop took place in the Georgia Institute of Technology (Savannah, Georgia, USA) during February 20-22, 2008. The workshop had a focus on Imprecise Probability in Engineering Analysis and Design, and its role in Reliable Engineering Computing; the meeting offered a unique combination of computer science, mathematics, and engineering. Several papers presented in the workshop have direct interest to the community. Detailed information, including a four-page booklet with all presented papers, can be found at www.gtsav.gatech.edu/workshop/rec08/.

Workshop on Principles and Methods of Statistical Inference with Interval Probability

This workshop took place in the Department of Mathematical Sciences, Durham University, during May 12-16 2008. The workshop was a joint effort with the interval probability research group of Ludwig-Maximilians University of Munich. The first day of the workshop was filled with presentations for a general audience; the other four days focused on specific topics (regression methods, Markov theory, decision making, and principles of statistics). Detailed information is available at www.maths.dur.ac.uk/users/matthias.troffaes/workshopip2008/.

Fourth International Workshop on Soft Methods in Probability and Statistics (SMPS08)

This workshop took place in Toulouse, France, during September 8-10, 2008. The

workshop, following similar events in 2002, 2004 and 2006, focused on approaches extending or orthogonal to the standard theory of probability and mathematical statistics. Various such approaches have appeared, either on their own like fuzzy set theory, possibility theory, rough sets, or having their origin in probability theory itself, like imprecise probability, belief functions, fuzzy random variables. Detailed information, including the list of accepted papers, can be found at www.irit.fr/smps08/.

WAET-Workshop 2008

This workshop took place at Ghent University (Ghent, Belgium), on Fri 19 Dec 2008. The workshop, organized within the SYSTeMS group at Ghent University, brought together researchers interested in imprecise probabilities, to attend six sessions of one hour. Thomas Augustin, Frank Coolen, Gert de Cooman, Filip Hermans, Erik Quaeghebeur, and Matthias Troffaes discussed desirability and utility, statistical inference, credal networks and Markov chains.

Symposium on Uncertainty Modeling of Rare/Imprecise Data

This mini-symposium with special sessions on uncertainty modeling of rare/imprecise data will happen at the 10th International Conference on Structural Safety and Reliability (ICOSSAR2009), to be held in Osaka, Japan, September 13-17 2009. Note that submissions are already closed; detailed information is available at http://www.sc.kutc.kansai-u.ac.jp/icossar2009/.

Second International Conference on Uncertainty in Structural Dynamics

This conference will take place at the University of Sheffield, UK, on June 15-17 2009. The conference is motivated by the concern that, as modern engineering relies increasingly on computer simulations and modeling for a wide range of processes, the modeling process must be robust to uncertainty. Detailed information is available at www.dynamics.group.shef.ac.uk/usd2009.

The 2008 SIPTA School

The goal of the SIPTA School, organized every even year since 2004, is to introduce interested students to the basics of imprecise probability topics, both theoretical and applied. The 2008 SIPTA School took place in Montpellier, France, during July 2-8 2008. Montpellier is a wonderful city, with a variety of attractions; located near the Mediterranean sea, it is close to many beautiful cities and beaches.

The School was organized by Kevin Loquin, Céline Berger, Olivier Strauss and Caroline Imbert, all working at the Laboratoire d'Informatique, de Robotique et de Microélectronique de Montpellier (LIRMM). The technical program was organized by Jean-Marc Bernard (researcher at CNRS and University of Paris Descartes) and Kevin Loquin.

The organization of the School was superb, to say the least. Lectures were given at the Centre Régional de Documentation Pédagogique (CRDP), with excellent facilities. Plenty of material was handed to participants during the School. Additionally, detailed information was (and still is) available at the web site www.lirmm.fr/SIPTASchool08/. In fact, this web site contains many photos taken during the School, and even videos with some of the lectures! Figure 1 shows one of these photos. Besides the lectures, there were excursions, visits to historic sites, banquets and gala dinners — with terrific French food. On top of this, the organizers demonstrated deep understanding of the French and European funding infrastructure: by extracting considerable resources from these entities, they could provide generous meals and discounts at hotels.

The following lectures were presented at the School (detailed abstracts can be found at the School's web site):

- **A unified view of uncertainty theories**
  by Didier Dubois (Université Paul Sabatier, Toulouse, France).

- **Coherent lower previsions I and II**
  by Enrique Miranda (Rey Juan Carlos University, Madrid, Spain) and Gert de Cooman (Ghent University, Zwijnaarde, Belgium).

- **Credal networks: Theory and applications**
  by Cassio P. de Campos (Rensselaer Polytechnic Institute, Troy, USA) and Fabio G. Cozman (Universidade de Sao Paulo, Sao Paulo, Brazil).

Figure 1: SIPTA School: A happy group of participants at the Place de la Comédie, Montpellier.

- **Algorithms and approximation methods for imprecise probability**
  by Fabio Cozman and Cassio P. de Campos.
- **Independence concepts in imprecise probability**
  by Fabio Cozman.
- **Predictive inference: from Bayesian inference to imprecise probability**
  by Jean-Marc Bernard (Laboratoire de Psychologie Environnementale, Boulogne-Billancourt, France).
- **Imprecise immediate predictions**, by Gert De Cooman (helped by Filip Hermans and Erik Quaeghebeur).
- **Robust Bayesian analysis**
  by Fabrizio Ruggeri (Istituto di Matematica Applicata e Tecnologie Informatiche, Milan, Italy).
- **What is risk? What is probability? Game-theoretic answers**
  by Glenn Shafer (Rutgers Business School, Newark, USA; also University of London, London, UK).

The lectures happened without major incidents. However, it must be noted that one particular lecture, on independence concepts in imprecise probability, gradually descended into chaos, with the lecturer apparently under the effect of too much French wine; sentences with esoteric terms such as "graphoids" and "E-admissibility" were heard by the dazed audience, and participants, dumbfounded as the speaker tried connections between non-pairwise preferences and exchangeability, were left speechless at the end of the talk. Amazing!

Abstracts on Imprecise Probabilities

This section lists papers announced at the SIPTA mailing list. Hopefully this is a valid incentive for people to announce their papers to the community!

Gert de Cooman, Filip Hermans, and Erik Quaeghebeur. **Imprecise Markov chains and their limit behaviour.**
arXiv:0801.0980 (http://arxiv.org/abs/0801.0980)
Abstract:
When the parameters of a finite Markov chain in discrete time, i.e., its initial and transition probabilities, are not well known, we can and should perform a sensitivity analysis. This is done by considering as basic uncertainty models the so-called credal sets that these probabilities are known or believed to belong to, and by allowing the probabilities to vary over such sets. This leads to the definition of an imprecise Markov chain. We show that the time evolution of such a system can be studied efficiently using so-called lower and upper expectations, which are equivalent mathematical representations of credal sets. We also study how the inferred credal set about the state at time n evolves as n goes to infinity, and we show that under quite unrestrictive conditions, this credal set converges to a uniquely invariant credal set, re-

gardless of the credal set given for the initial state of the system. We thus effectively prove a Perron-Frobenius Theorem for a special class of non-linear dynamical systems in discrete time.

Gert de Cooman and Filip Hermans. **Imprecise probability trees: Bridging two theories of imprecise probability.**
arXiv:0801.1196 http://arxiv.org/abs/0801.1196)
Abstract:
We give an overview of two approaches to probability theory where lower and upper probabilities, rather than probabilities, are used: Walley's behavioural theory of imprecise probabilities, and Shafer and Vovk's game-theoretic account of probability. We show that the two theories are more closely related than would be suspected at first sight, and we establish a correspondence between them that (i) has an interesting interpretation, and (ii) allows us to freely import results from one theory into the other. Our approach leads to an account of probability trees and random processes in the framework of Walley's theory. We indicate how our results can be used to reduce the computational complexity of dealing with imprecision in probability trees, and we prove an interesting and quite general version of the weak law of large numbers.

Gert de Cooman and Enrique Miranda. **Symmetry of models versus models of symmetry.**
arXiv:0801.1966v1
(http://arxiv.org/abs/0801.1966v1)
Abstract:
A model for a subject's beliefs about a phenomenon may exhibit symmetry, in the sense that it is invariant under certain transformations. On the other hand, such a belief model may be intended to represent that the subject believes or knows that the phenomenon under study exhibits symmetry. We defend the view that these are fundamentally different things, even though the difference cannot be captured by Bayesian belief models. In fact, the failure to distinguish between both situations leads to Laplace's so-called Principle of Insufficient Reason, which has been criticised extensively in the literature. We show that there are belief models (imprecise probability models, coherent lower previsions) that generalise and include the Bayesian belief models, but where this fundamental difference can be captured. This leads to two notions of symmetry for such belief models: weak invariance (representing symmetry of beliefs) and strong invariance (modelling beliefs of symmetry). We discuss various mathematical as well as more philosophical aspects of these notions. We also discuss a few examples to show the relevance of our findings both to probabilistic modelling and to statistical inference, and to the notion of exchangeability in particular.

Gert de Cooman, Erik Quaeghebeur, and Enrique Miranda. **Exchangeable lower previsions.**
arXiv:0801.1265v1
(http://arxiv.org/abs/0801.1265v1)
Abstract:
We extend de Finetti's (1937) notion of exchangeability to finite and countable sequences of variables, when a subject's beliefs about them are modelled using coherent lower previsions rather than (linear) previsions. We prove representation theorems in both the finite and the countable case, in terms of sampling without and with replacement, respectively. We also establish a convergence result for sample means of exchangeable sequences. Finally, we study and solve the problem of exchangeable natural extension: how to find the most conservative (point-wise smallest) coherent and exchangeable lower prevision that dominates a given lower prevision.

Gert de Cooman, Matthias C. M. Troffaes, and Enrique Miranda. **n-Monotone exact functionals.**
arXiv:0801.1962v1
http://arxiv.org/abs/0801.1962v1)
Abstract:
We study n-monotone functionals, which constitute a generalisation of n-monotone set functions. We investigate their relation to the concepts of exactness and natural extension, which generalise the notions of coherence and natural extension in the behavioural theory of imprecise probabilities. We improve upon a number of results in the literature, and prove among other things a representation result for exact n-monotone functionals in terms of Choquet integrals.

G. Corani and M. Zaffalon. **Learning reliable classifiers from small or incomplete data sets: the naive credal classifer 2.**
Journal of Machine Learning Research 9(Apr):581–621, 2008.
(http://jmlr.csail.mit.edu/papers/v9/corani08a.html)

F. G. Cozman. **Concentration inequalities and laws of large numbers under irrelevance of lower and upper expectations.**
arXiv:0810.2821v1
http://arxiv.org/abs/0810.2821v1)
Abstract:

This paper presents concentration inequalities and laws of large numbers under weak assumptions of irrelevance, expressed through lower and upper expectations. The results extend de Cooman and Miranda's recent results concerning epistemic irrelevance. The proofs indicate connections between concepts of irrelevance for lower/upper expectations and the standard theory of martingales.

Algorithms for Interval Uncertainty

by Gang Xiang[1]
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Formulation of the problem.

Statistical analysis and, more generally, statistical data processing is an extremely important part of modern science and engineering. It is therefore necessary to use the most adequate statistical techniques. Many traditional statistical formulas $y = C(x_1, \ldots, x_n)$, e.g., the formulas for the population mean, population variance, etc., are based on the simplifying assumptions that for some sample, we know the exact values $x_1, \ldots, x_n$ of the quantity of interest. In reality, these values usually come from measurements or from expert estimates; both methods are not absolutely accurate. It is thus desirable to find out how the inaccuracy with which we know the sample values affects the result of statistical analysis.

In many real-life situations, we only know the upper bound Δ on the measurement inaccuracy. In this case, once we know the result $\widetilde{x}$ of measuring the desired quantity, the only information that we have about the actual (unknown) value x of this quantity is that this value must belong to the interval $[\widetilde{x} - \Delta, \widetilde{x} + \Delta]$. Thus, it is necessary to generalize traditional statistical formulas $C(x_1, \ldots, x_n)$ to the case when we only know the inputs with interval uncertainty. In precise terms, we know n intervals $\mathbf{x}_1, \ldots, \mathbf{x}_n$, and we would like to compute the range $\mathbf{C}$ of possible values of the given characteristic $C(x_1, \ldots, x_n)$ when $x_i \in \mathbf{x}_i$.

In general, this computational problem is NP-hard even for the variance. However, in some practically important situations, it has been possible to design efficient algorithms for computing these characteristics. The main objective of this research was to improve the existing algorithms and to design new algorithms for practically important situations which were not covered by the previous algorithms.

Main results.

We reduce the computational complexity of several known efficient algorithms to the complexity $O(n \cdot \log(n))$ comparable with the complexity of simply sorting the given values, and in some situations, even to the linear time complexity $O(n)$ – the smallest possible computational complexity of any algorithm for processing n input values $x_1, \ldots, x_n$. We also designed new efficient algorithms for computing variance and other important statistical characteristics for several practically important classes of statistical problems.

Specifically, for the lower endpoint $\underline{V}$ for the variance V, the fastest previously known algorithm takes $O(n \cdot \log(n))$ time. We have developed a linear time algorithm for computing $\underline{V}$.

The problem of computing the upper endpoint $\overline{V}$ is, in general, NP-hard. Previously, $O(n^2)$ time algorithms were known for the cases of narrow intervals (when no two intervals intersect), of slightly wider intervals (when for some K, no group of K intervals has a common point), and of privacy (when every two intervals either coincide or do not intersect). We reduced the computational complexity of computing $\overline{V}$ to linear time for narrow intervals and for the privacy case, and to $O(n \cdot \log(n))$ times for slightly wider intervals. We also designed a new linear time algorithm for the case of a single measuring instrument MI (when no interval is a proper subset of another one), and an $O(n^m)$ time algorithm for the case of m measuring instruments, when input intervals can be divided into m classes each of which satisfies the above single-MI no-subset property.

We produced similar results for estimating the range of the endpoints $L = E - k_0 \cdot \sigma$ and $U = E + k_0 \cdot \sigma$ of the confidence interval which is often used to detect outliers (with $k_0 = 2$, $k_0 = 3$, or $k_0 = 6$). For the cases of narrow and slightly wider intervals, we reduced the computation time from $O(n^2)$ to $O(n \cdot \log(n))$. We also designed a new $O(n \cdot \log(n))$ time algorithm for the single MI case and a new $O(n^m)$ time algorithm for the case of m MI.

We also describe efficient algorithms for com-

[1] A summary of results from the PhD thesis by the author, advised by Prof. Vladik Kreinovich and defended in 2007 at the Department of Computer Science, University of Texas at El Paso — El Paso, TX 79968, USA.

puting the range of another important statistical characteristics – skewness (third central moment), a characteristic important for describing possible asymmetry of the probability distributions.

For all these algorithms, their correctness has been proven.

Applications.

Our algorithms were motivated by (and applied to) several applied problems.

In *computer security*, we deal with an interval method of preserving privacy in which for each sensitive field of data, instead of the actual values of the corresponding quantity, we only keep a range (interval) of possible values of this quantity: e.g., instead of the exact age of 28, we only keep a range $[20, 30]$. For this application, we have developed new efficient algorithms for estimating values of different statistical characteristics under such privacy-related interval uncertainty.

In *geosciences*, we deal with the inverse problem of geophysics, where we measure the seismic signals generated by artificial small explosions, and then reconstruct the velocities at different 3-dimensional points from the travel times of this seismic signal. In this problem, the existing algorithms often produce the velocities v which are outside the geophysically known intervals $[\underline{v}, \overline{v}]$ of possible values. We showed how to take this interval information into account when solving the inverse problems.

In *computer engineering*, we deal with the problem of estimating the clock cycle in chip design. Traditional methods for clock cycle estimation only consider interval (worst-case) uncertainty. In reality, we often also have additional information about the mean values of the delays in the corresponding gates and wires. We showed how this information can be used to provide more realistic estimates for the clock cycle and thus, improve the efficiency of the chips.

Finally, in 1-dimensional *radar data processing*, we show how we can combine the probabilistic uncertainty corresponding to measurement errors and interval uncertainty corresponding to finite distance resolution and thus distinguish the explosion core from the explosion fragments.

More details...

...available from the author's dissertation at www.cs.utep.edu/vladik/xiang.pdf. Most papers are available from the author's website.

Software section: The *DecideIt* tool

The following text is a message posted at the SIPTA mailing list by Love Ekenberg, one of the key researchers behind DecideIt.

The decision analytic tool DecideIT is one of the few tools available that can model interval probabilistic and multi-criteria decision problems in the same model. The tool was developed by the Decide Research Group, a research network at several Swedish universities with the main node at Stockholm University and the Royal Institute of Technology.

DecideIT was designed based on experience with a large number of commercially available decision analytic tools. The input data is only required to be as precise as the information available, since, as we all know, in most real-life cases, only data with some imprecision is available, for example in the form of estimated probability, utility and cost intervals.

The tool was built around the idea that sensitivity analyses should be integrated into the representation, thus permitting sensitivity to become an integral part of the decision evaluation. Distinguishing features of the tool include:

- Seamlessly modelling probabilistic and multi-criteria decision problems together.
- Allowing imprecision in the input data in the form of intervals and comparisons.
- Intervals, comparisons, and fixed numbers can be mixed quite arbitrarily.
- Threshold handling by security levels.
- Several complementary evaluation methods with different levels of detail.
- Fast algorithms allowing the use of thousands of consequences.

The current version of DecideIT has been developed by a spin-off company, Preference, owned by Stockholm University and the Royal Institute of Technology in addition to the main researchers of the Decide Research Group.

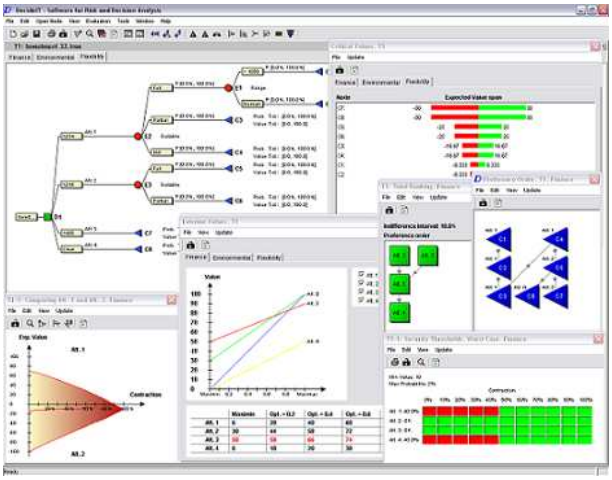

Figure 2: A screenshot of the DecideIt tool.

The tool is now made available to academic researchers and teachers all over the world FREE OF CHARGE. This offer is valid for academic purposes only. You will find all necessary information at www.preference.nu/site_en/purchase.php

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