Efficient reliability analysis with imprecise probabilities

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Thanks to all colleagues involved in this work:
Matteo Broggi, Pengfei Wei, Jingwen Song, Marcos Valdebenito, Matthias Faes, Cao Wang, Hao Zhang, Sifeng Bi, Yi Zhang, Edoardo Patelli, Marco deAngelis, ...

ENGINEERING ANALYSIS

Endeavor

- numerical modeling – physical phenomena, structure, and environment

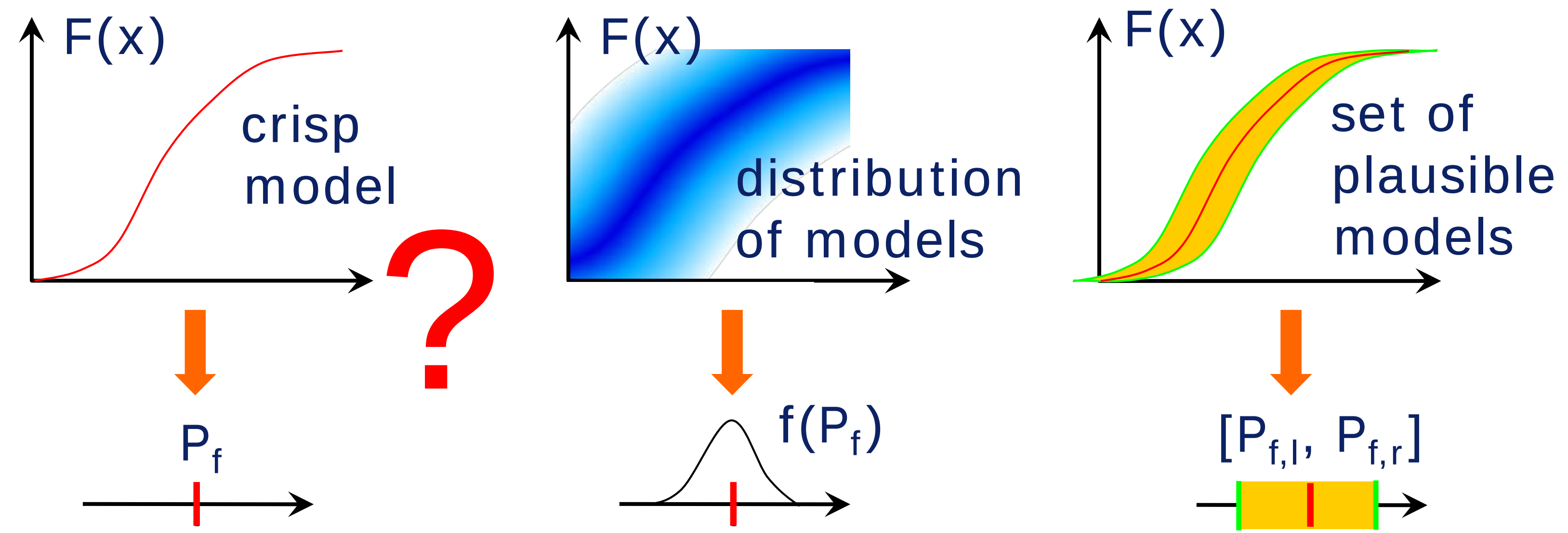
  → prognosis – system behavior, hazards, safety, risk, robustness, economic and social impact, ...

» close to reality
» numerically efficient

↔

Challenge: Uncertainties

Aleatory and epistemic uncertainty



CONCEPT OF MODELS AND PROCESSING

Hybrid uncertainties, imprecise probabilities

- probabilistic models with set-valued descriptors
- set-valued system description
- parametric or non-parametric descriptions

➡ **bounding probabilities of events of interest** (in association with some confidence level)

Probability boxes (p-boxes)

- set of distribution functions (i.e. set of random variables) $\tilde{F}(x) = \left\{ F_j(x) \;\middle|\; F_j(x) \in \left[F_l(x), F_u(x) \right] \; \forall x \right\}$

Fuzzy probabilities

- fuzzy set of p-boxes

$$\tilde{F}(x) = \left\{ \left(F_\alpha(x), \mu\left(F_\alpha(x)\right) \right) \;\middle|\; F_\alpha(x) = \left[F_{\alpha l}(x), F_{\alpha u}(x) \right], \; \mu\left(F_\alpha(x)\right) = \alpha \;\; \forall \alpha \in (0,1] \right\}$$

Numerical processing

- stochastic techniques combined with interval / fuzzy analysis techniques

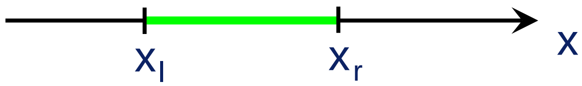
Beer, M.; Ferson, S.; Kreinovich, V. (2013):
Imprecise Probabilities in Engineering Analyses, Mechanical Systems and Signal Processing 37, 4–29.

Beer, M.; Zhang, Y.; Quek, S.T.; Phoon, K.K. (2013):
Reliability analysis with scarce information: Comparing alternative approaches in a geotechnical engineering context, Structural Safety 41, 1–10.

SET-THEORETICAL DESCRIPTORS – IMPRECISION

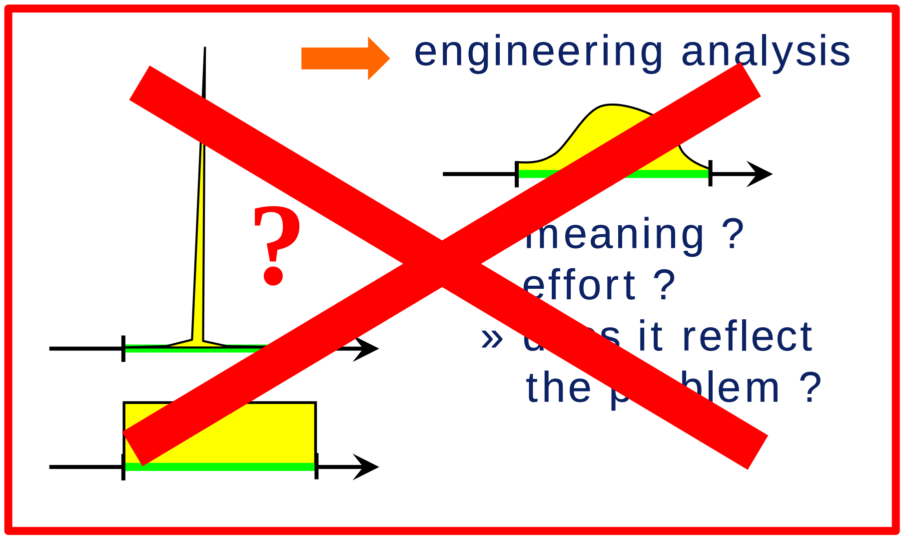
Interval

- $X = [x_l, x_r] = \{\ x \in \mathbf{X} = \mathbb{R}\ |\ x_l \le x \le x_r\ \}$



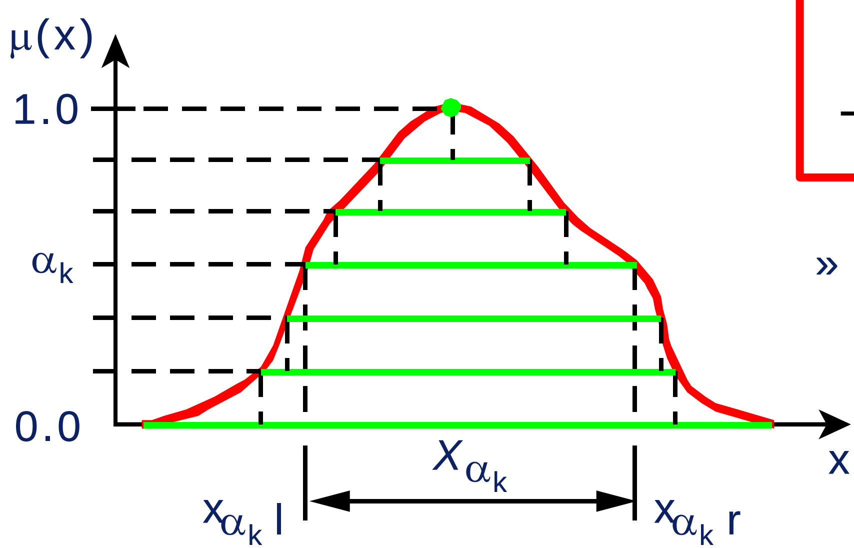
» possible value range between crisp bounds

» no additional information



Fuzzy sets

- α-level set $X_\alpha = \{\ x \in \mathbf{X}\ |\ \mu(x) \ge \alpha\ \}$
- α-discretization $\tilde{X} = \{(X_\alpha, \mu(X_\alpha))\}$



» set of nested intervals of various size

➡ instrument to explore influence of interval size (sensitivity wrt. epistemic uncertainty) in an intuitive and structured manner

Möller, B.; Graf, W.; Beer, M. (2000):
Fuzzy structural analysis using α-level optimization, Computational Mechanics 26, 547–565.

ANALYSIS WITH INTERVALS AND FUZZY SETS

Naive approach: nested analysis, double/triple loop

repeated interval analysis → fuzzy set (results for varying interval size)

repeated stochastic analysis → interval bounds

repeated deterministic systems or structural analysis

Goal: calculate fuzzy / interval result
from single efficient stochastic analysis

Interval arithmetic

- implementation of interval-valued variables in numerical algorithm
  - » intrusive
  - » requires intrinsic reformulation of algorithm to minimize dependability problem
  - » narrow actual result interval from outside, tightest enclosure
  - » restricted to the very specific problem classes

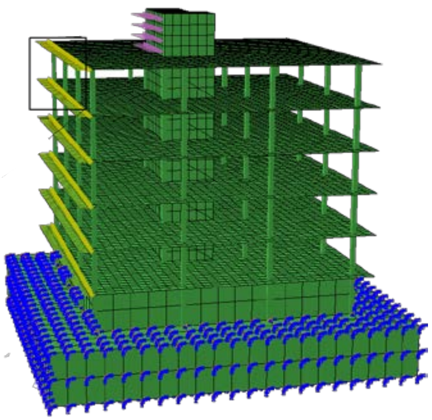
Optimization approaches

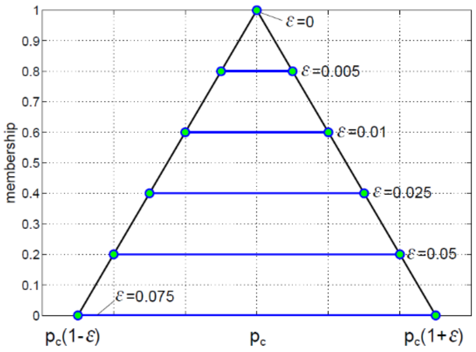
- explicit search for result interval bounds
  - » intrusive: reformulation of problem structure
    - ▪ to exploit problem topology
    - ▪ to utilize linear algebra or linear programming approaches
  - » non-intrusive: model order reduction, surrogate problem representation, sampling-based solution
  - » applicable to large variety of problems

RELIABILITY ANALYSIS IN HIGH DIMENSIONS

Multi-storey building – reliability for component failure

- structure
  - » 8,200 finite elements, 66,300 dof
- imprecise probabilistic input
  - » 488 fuzzy parameters for 244 fuzzy random variables





$\pm$ 7.5 % tolerance range

SV #	Prob. dist.	$\underline{p} = p_c\ [1-\epsilon,\ 1+\epsilon]$		Description	Units
1	$N(\underline{\mu},\ \underline{\sigma})$	$\mu_c = 0.1$	$\sigma_c = 0.01$	Columns' strength	GPa
2 – 193	$Unif(\underline{a}, \underline{b})$	$a_c = 0.36$	$b_c = 0.44$	Sections' size	m
194 – 212	$LN(\underline{m},\ \underline{v})$	$m_c = 35$	$v_c = 12.25$	Young's modulus	GPa
213 – 231	$LN(\underline{m},\ \underline{v})$	$m_c = 2.5$	$v_c = 6.25\ 10^{-2}$	Material's density	kg/dm^3
232 – 244	$LN(\underline{m},\ \underline{v})$	$m_c = 0.25$	$v_c = 6.25\ 10^{-4}$	Poisson's ratio	-

ADVANCED LINE SAMPLING, ROBUST RELIABILITY

Retrieving optimal points from problem topology

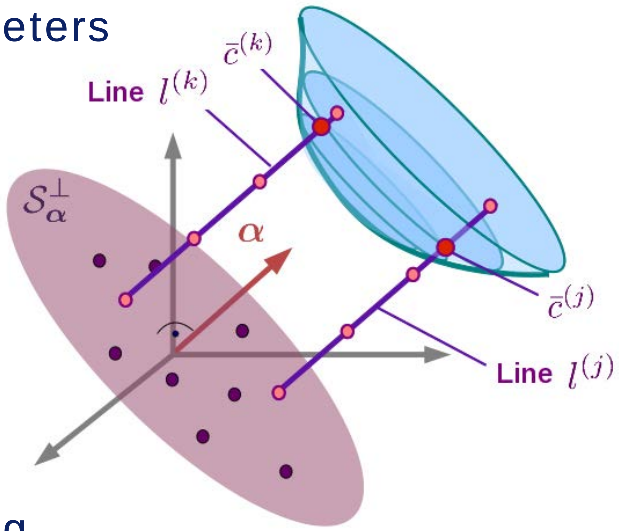
- global optimization problem

$$\underline{p_f} = \inf_{x,p} \int_{\Omega_f(x)} h_d(\xi, p)\, d\Omega$$

$$\overline{p_f} = \sup_{x,p} \int_{\Omega_f(x)} h_d(\xi, p)\, d\Omega$$

p – distribution parameters
ξ – random variables
x – intervals

Ω_f depends on intervals x **!**



- map intervals x to augmented probability space

$$\Omega \times X \to \Theta : \quad x \to \eta \in \mathbb{C}_x = \left\{ h_n\left(\eta; \underline{\overline{\mu_x}}, \sigma_x\right) \middle| \underline{\overline{\mu_x}} = \underline{\overline{x}} \right\}$$

- exploit topological properties of Θ for line sampling

sampling direction $-\nabla g$

➡ optimal points $\left(p^u, x^u\right) = \psi^u\left(-\nabla g\right), \quad \left(p^l, x^l\right) = \psi^l\left(-\nabla g\right)$

➡ $\overline{p_f} = \int_{\Omega_f(x^u)} h_d\left(\xi, p^u\right) d\Omega, \qquad \underline{p_f} = \int_{\Omega_f(x^l)} h_d\left(\xi, p^l\right) d\Omega$

$$\hat{P}_f = \frac{1}{N_L} \sum_{i=1}^{N_L} \phi\left(-\bar{c}^{(i)}\right)$$

distributed computing

De Angelis, M.; Patelli, E.; Beer, M. (2015):
Advanced Line Sampling for Efficient Robust Reliability Analysis, Structural Safety 52, 170–182.

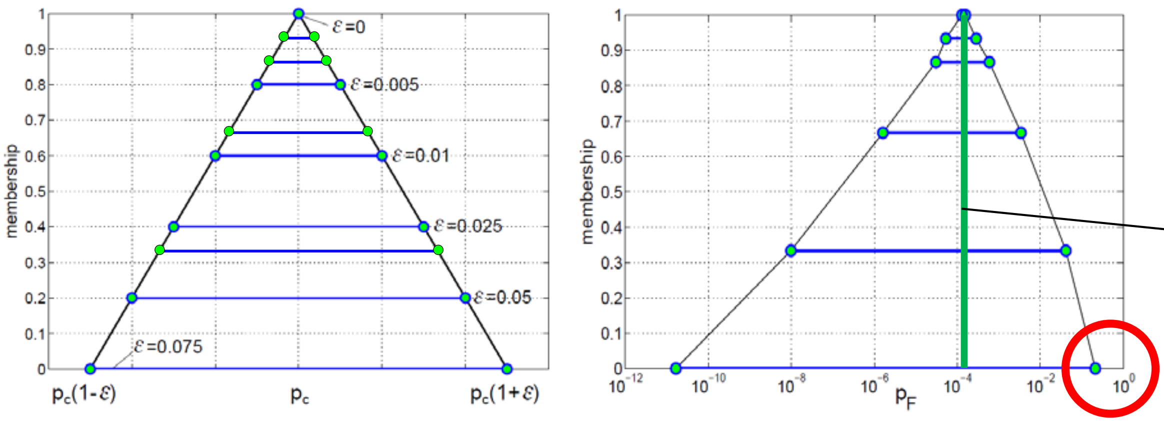
ADVANCED LINE SAMPLING, ROBUST RELIABILITY

Multi-storey building – results

- advanced line sampling with pre-identified optimal points in Θ

	Lower Bound		Upper Bound		
ϵ	$\underline{p}_F$	CoV	$\overline{p}_F$	CoV	Ns
0.000	$1.42\ 10^{-4}$	$9.2\ 10^{-2}$	$1.42\ 10^{-4}$	$9.2\ 10^{-2}$	126
0.005	$5.75\ 10^{-5}$	$8.7\ 10^{-2}$	$2.63\ 10^{-4}$	$7.1\ 10^{-2}$	257
0.010	$4.57\ 10^{-5}$	$33.6\ 10^{-2}$	$5.30\ 10^{-4}$	$11.5\ 10^{-2}$	250
0.025	$1.75\ 10^{-6}$	$8.8\ 10^{-2}$	$3.22\ 10^{-3}$	$5.3\ 10^{-2}$	253
0.050	$2.27\ 10^{-8}$	$57.0\ 10^{-2}$	$3.88\ 10^{-2}$	$5.4\ 10^{-2}$	255
0.075	$1.88\ 10^{-11}$	$12.2\ 10^{-2}$	$2.02\ 10^{-1}$	$3.5\ 10^{-2}$	254

sample size 1,395



sensitivity of failure probability

$P_f \approx 10^{-4}$

safety alerts

$\approx 10^{-1}$!

TIME DEPENDENT RELIABILITY ANALYSIS

First passage problem

- pre-identification of $\bar{\theta}^*$ such that

$$\bar{P}_f = \int_{z \in \mathbb{R}^n} I_F\left(z, \bar{\theta}^*\right) f_Z(z)\, dz$$

with

Operator norm

$$\bar{\theta}^* = \underset{\theta \in \theta^I}{\arg\max} \; \max_{i=1,\dots,n_y} \; \max_{l} \left\| A_{i,l}(\theta) \right\|_2$$

via standard optimization
on the physical model (ie FEM)
without repeated reliability analysis

- requirement:
find a continuous linear map A
that relates random input z
to random response y

$$y(\theta) = A(\theta) z$$

- operator norm theory

$$\left\| A_i(\theta) z \right\|_{p^{(1)}} \le \left| c_i(\theta) \right| \cdot \left\| z \right\|_{p^{(2)}}$$

$$\left\| y_i(t, \theta, z) \right\|_{p^{(1)}} \le \left| c_i(\theta) \right| \cdot \left\| z \right\|_{p^{(2)}}$$

→ smallest $|c_i(\theta)|$ provides
upper bound on „amplification“

» $p^{(1)} = \infty$: focus on largest response
to retrieve first excursion

» $p^{(2)} = 2$: to relate to
energy content of load

$$\left\| A \right\|_{p^{(1)}, p^{(2)}} = \max_{l} \left\{ \frac{\left\| A_i(\theta) z \right\|_{p^{(1)}}}{\left\| z \right\|_{p^{(2)}}} \right\}$$

$$\left\| A \right\|_{p^{(1)}, p^{(2)}} = \max_{l} \left\| A_{i,l}(\theta) \right\|_2$$

Faes, M.; Valdebenito, M.A.; Moens, D.; Beer, M. (2020):
Bounding the First Excursion Probability of Linear Structures Subjected to Imprecise Stochastic Loading, Computers and Structures 239, 106320.

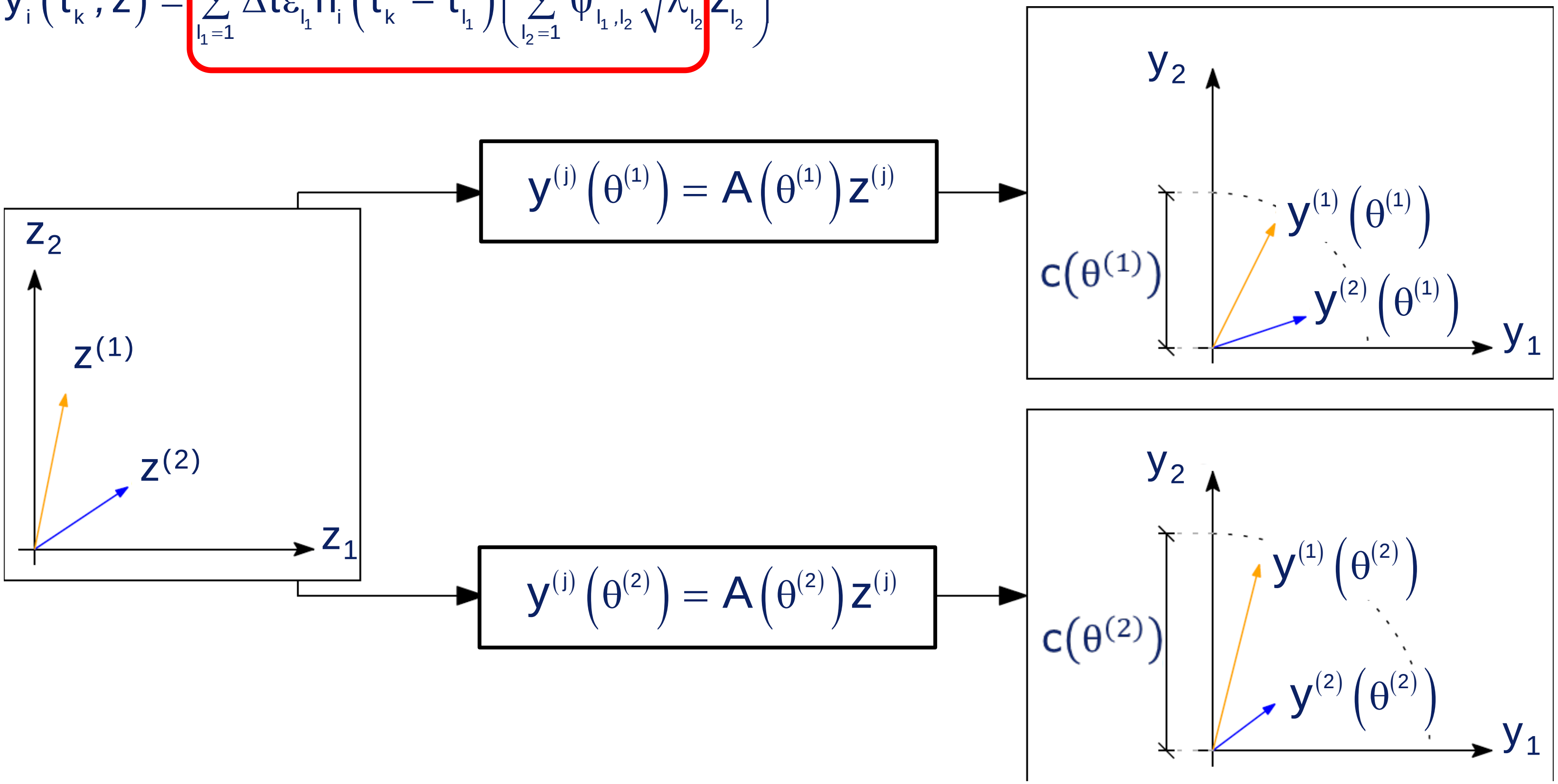
Faes, M.; Valdebenito, M.A.; Moens, D.; Beer, M. (2021):
Operator norm theory as an efficient tool to propagate hybrid uncertainties and calculate imprecise probabilities,
Mechanical Systems and Signal Processing 152, 107482.

TIME DEPENDENT RELIABILITY ANALYSIS

First passage problem

- Karhunen-Loeve expansion

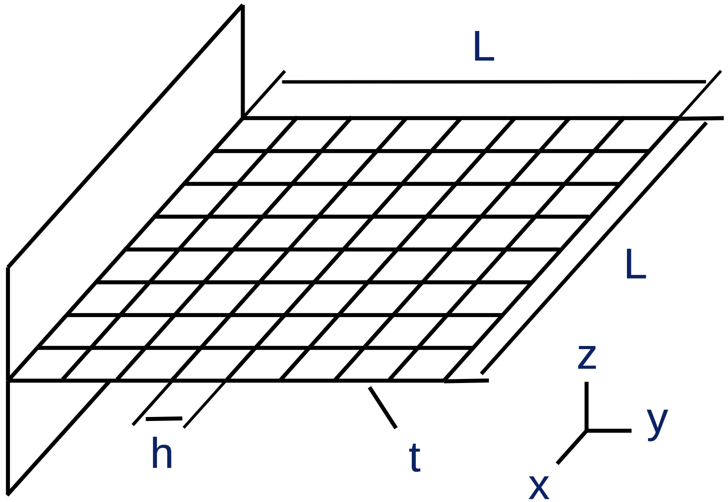
$$y_i\left(t_k, z\right) = \underbrace{\sum_{l_1=1}^{k} \Delta t \varepsilon_{l_1} h_i\left(t_k - t_{l_1}\right)\left(\sum_{l_2=1}^{n_{KL}} \psi_{l_1, l_2} \sqrt{\lambda_{l_2}}}_{A_i} z_{l_2}\right)$$



TIME DEPENDENT RELIABILITY ANALYSIS

Example: clamped steel plate

- structural model
  - » 100 shell elements, linear
  - » 110 nodes
  - » Dirichlet boundary conditions on clamp



- P_f for exceedance of displacement at corner point of 15 cm

- load model

$$F(r,\theta,z) = 1 \cdot \theta_1 \cdot sin\left(\frac{\pi}{\theta_2}\right) + \theta_3 \cdot B(\theta_4, r) \cdot z$$

with
  - » KL-basis B
  - » exponential covariance kernel[1)]
  - » 10 standard normal rv's z

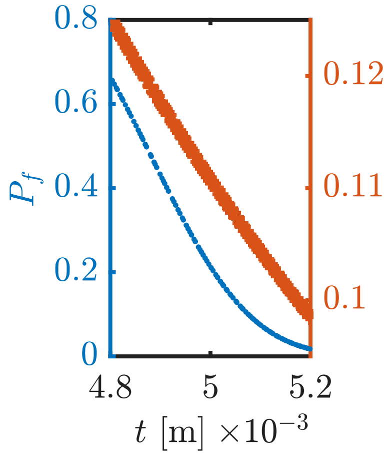
- interval parameters
  - » θ_1 and θ_2 governing the expected value of random load field
  - » θ_3: standard deviation of load field
  - » θ_4: correlation length of load field
  - » E: Young's modulus
  - » t: plate thickness

1) Note: Faes, M.G.R.; Broggi, M.; Spanos, P.D.; Beer, M. (2022): Elucidating appealing features of differentiable auto-correlation functions: a study on the modified exponential kernel, Probabilistic Engineering Mechanics, 69, 103269. Spanos, P.D.; Beer, M.; Red-Horse, J. (2007): Karhunen-Loéve Expansion of Stochastic Processes with a Modified Exponential Covariance Kernel, ASCE Journal of Engineering Mechanics, 133(7), 773–779.

TIME DEPENDENT RELIABILITY ANALYSIS

Example: clamped steel plate

- dependencies between interval parameters, operator norm and P_f



TIME DEPENDENT RELIABILITY ANALYSIS

Example: clamped steel plate

- results and numerical efficiency
  - » particle swarm optimization to evaluate operator norm
  - » FORM to compute P_f (problem linear in z and low dimensionality)
  - » comparison with vertex method and double loop solution

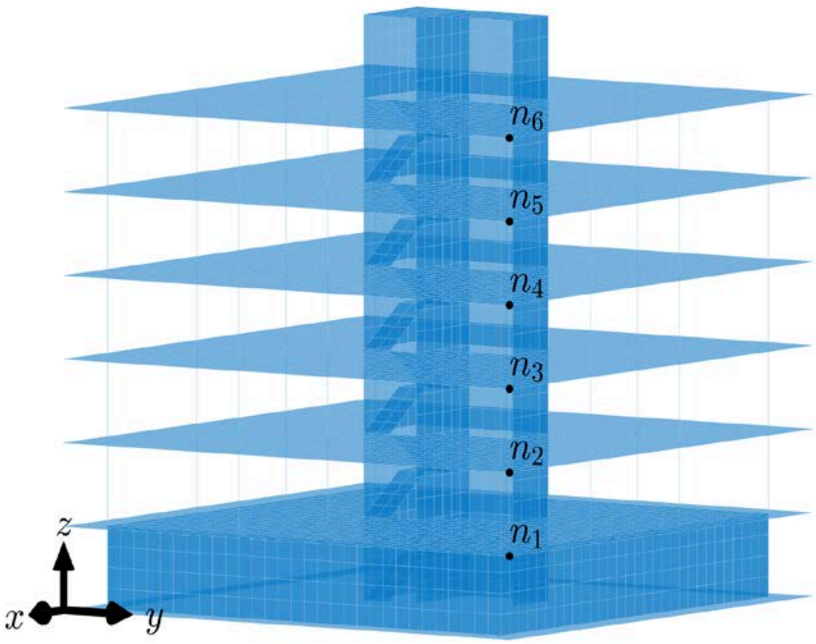
	vertex method		operator norm		double loop	
	$\underline{\theta}^{*}$	$\overline{\theta}^{*}$	$\underline{\theta}^{*}$	$\overline{\theta}^{*}$	$\underline{\theta}^{*}$	$\overline{\theta}^{*}$
operator norm	0.0208	0.0859	0.0208	0.1112	0.0208	0.1112
P_f	$8.67 \cdot 10^{-6}$	0.2907	$8.67 \cdot 10^{-6}$	0.4889	$8.67 \cdot 10^{-6}$	0.4889
FE analyses	1794		640+47	880+33	18156	26539

» numerical effort significantly reduced
» correct identification of internal optimal points

TIME DEPENDENT RELIABILITY ANALYSIS

Example: six-story building under earthquake excitation

- structural model
  - » 9500 shell and beam elements, linear
  - » reinforced concrete



- load model
  - » Gaussian stochastic process
  - » Autocorrelation governed by modulated Clough-Penzien spectrum
- interval parameters
  - » 7 parameters of the load model
  - » Young's modulus of concrete for each story

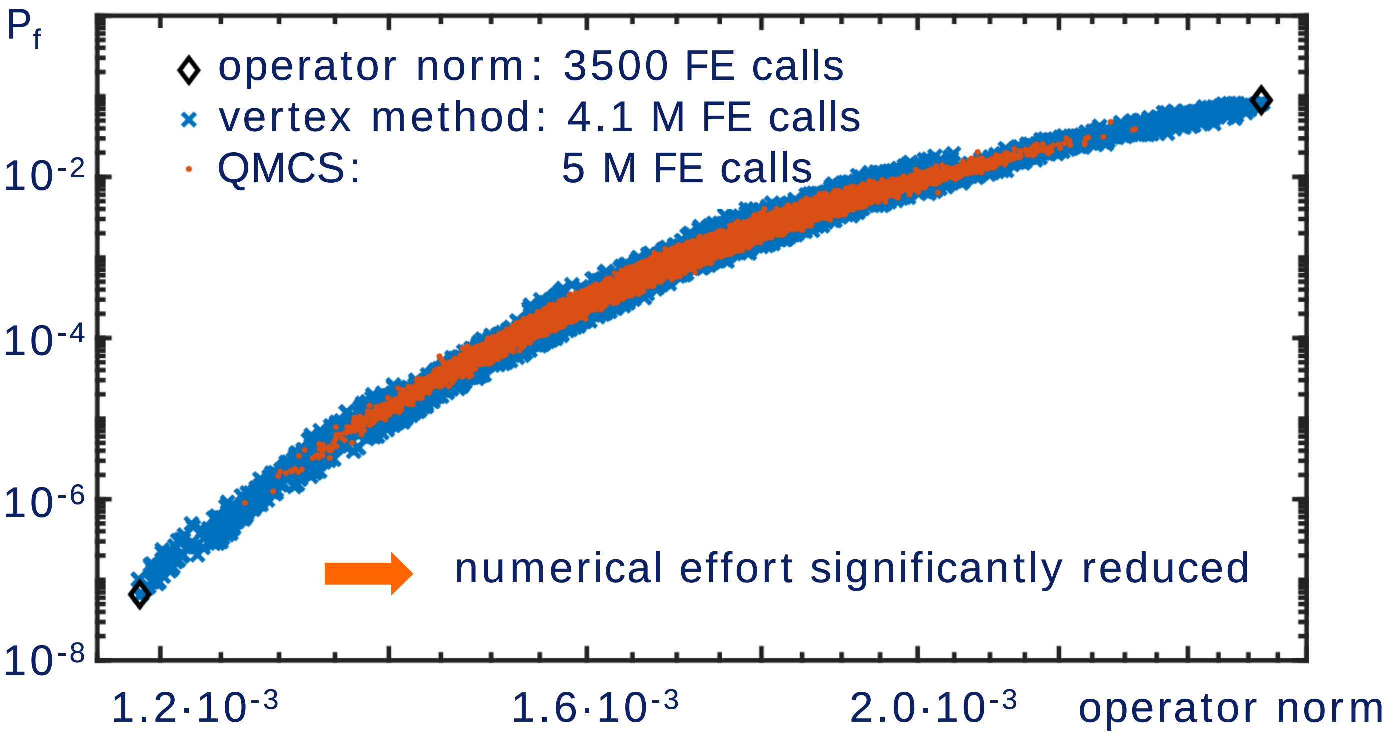
→ 13 interval parameters

- P_f for exceedance of interstory drift of $2 \cdot 10^{-3}$ times the story height

TIME DEPENDENT RELIABILITY ANALYSIS

Example: six-story building under earthquake excitation

- results and numerical efficiency
  - » particle swarm optimization to evaluate operator norm
  - » directional importance sampling to compute P_f
  - » comparison with vertex method and quasi MCS to explore intervals



RELIABILITY ANALYSIS OF LARGE STRUCTURES

Parametric surrogate with intervening variables

- failure probability and sensitivities

$$P_f(\theta) = \int_{x \in \mathbb{R}^n} I_F(x)\, f_X(x|\theta)\, dx$$

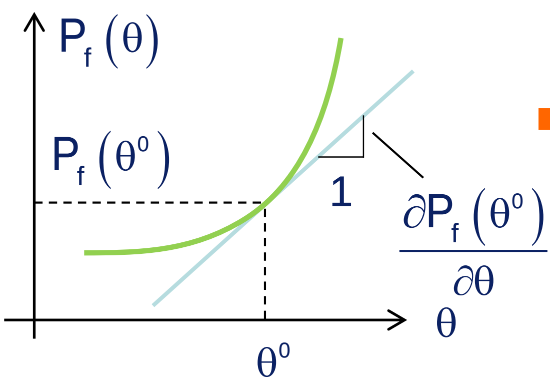
$$\frac{\partial P_f(\theta)}{\partial \theta_j} = \int_{x \in \mathbb{R}^n} I_F(x) \frac{\partial f_X(x|\theta)}{\partial \theta_j} dx$$

- estimation through sampling

$$\hat{P}_f(\theta) = \frac{1}{N} \cdot \sum_{i=1}^{N} I_F\left(x^{(i)}\right)$$

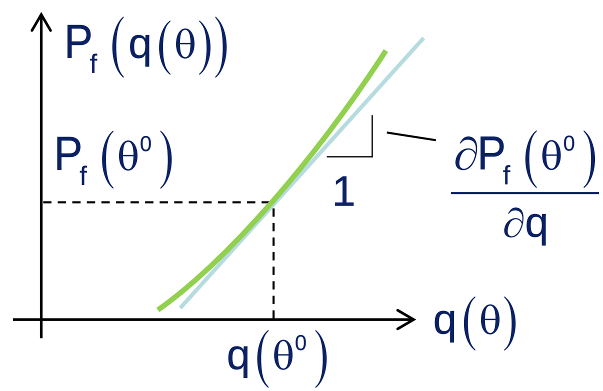
$$\widehat{\left(\frac{\partial P_f(\theta)}{\partial \theta_j}\right)} = \frac{1}{N} \cdot \sum_{i=1}^{N} I_F\left(x^{(i)}\right) \frac{\partial f_X\left(x^{(i)}|\theta\right) / \partial \theta_j}{f_X\left(x^{(i)}|\theta\right)}$$

- linear approximation w.r.t. θ



intervening variable $q(\theta)$ to capture nonlinearities

- linear approximation w.r.t. $q(\theta)$



Valdebenito, M.A.; Pérez, C.A.; Jensen, H.A.; Beer, M. (2016):
Approximate fuzzy analysis of linear structural systems applying intervening variables, Computers & Structures 162, 116–129.

Valdebenito, M.A.; Beer, M.; Jensen, H.A.; Chen, J.B.; Wei, P.F. (2020):
Fuzzy Failure Probability Estimation Applying Intervening Variables, Structural Safety 83, 101909.

RELIABILITY ANALYSIS OF LARGE STRUCTURES

Parametric surrogate with intervening variables

- choice of intervening variables – two level approximation

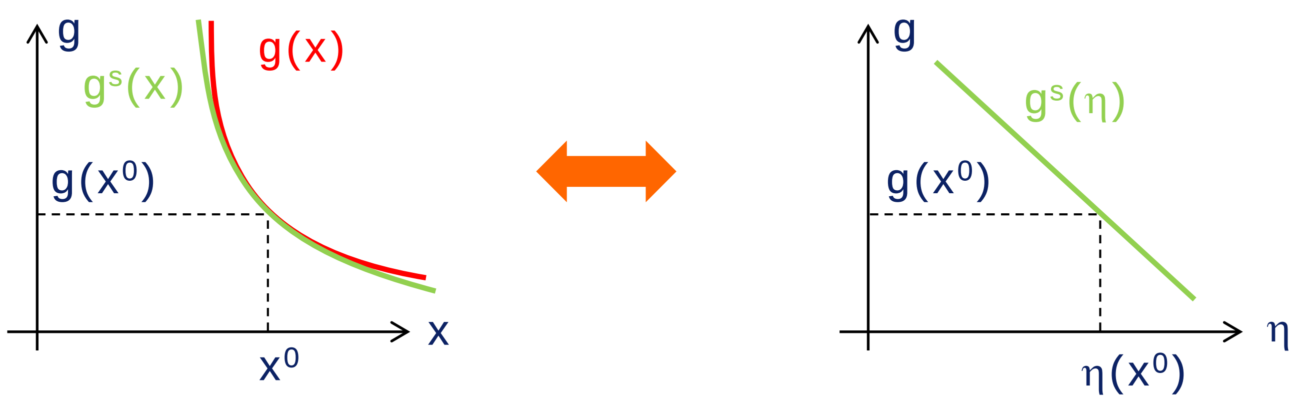
  (I) approximate representation of performance function

$$g(x) \approx g^s(x) = g_0 + \sum_{i=1}^{n_z} g_i \cdot \left(\eta_i(x_i) - \eta_i\left(x_i^0\right)\right) \quad \text{with} \quad \eta_i(x_i) = x_i^{m_i} \quad \text{(power type)}$$

  » find the $2n_x + 1$ coefficients g_i and m_i such that

$$g\left(x^0\right) = g^s\left(x^0\right), \quad \frac{\partial g\left(x^0\right)}{\partial x_i} = \frac{\partial g^s\left(x^0\right)}{\partial x_i}, \quad \frac{\partial^2 g\left(x^0\right)}{\partial x_i^2} = \frac{\partial^2 g^s\left(x^0\right)}{\partial x_i^2}$$

nonlinear approximation of g(x) corresponding linear function $g^s(\eta)$



RELIABILITY ANALYSIS OF LARGE STRUCTURES

Parametric surrogate with intervening variables

- choice of intervening variables – two level approximation
  - (II) approximate representation of failure probability

$$P_f(\theta) \approx P_f^s(\theta) = \Phi\left(h_0 + \sum_{j=1}^{n_\xi} h_j\left(\xi_j(\theta) - \xi_j(\theta^0)\right)\right)$$

$$h_0 = \Phi^{-1}\left(\hat{P}_f(\theta^0)\right)$$

h_j from $\widehat{\left(\frac{\partial P_f(\theta)}{\partial \theta_j}\right)}$

» inspired by FORM applied to $g^s(x)$

$$\xi_j(\theta) = \frac{\eta_j\left(t_j\left(z=0\,|\,\theta_j\right)\right)}{\sqrt{\sum_{i=1}^{n_x}\left(g_i\left(\frac{\partial \eta_i\left(t_i\left(z_i\,|\,\theta_i\right)\right)}{\partial z_i}\right)\Big|z_i=0\right)^2}}, \quad j = 1, \ldots, n_x$$

$\mu_j^{m_j}$ if X are normal

$g_i \cdot m_i \cdot \mu_i^{m_i - 1} \cdot \sigma_i$ if X are normal

$$\xi_j(\theta) = \frac{1}{\sqrt{\sum_{i=1}^{n_x}\left(g_i\left(\frac{\partial \eta_i\left(t_i\left(z_i\,|\,\theta_i\right)\right)}{\partial z_i}\right)\Big|z_i=0\right)^2}}, \quad j = n_x + 1$$

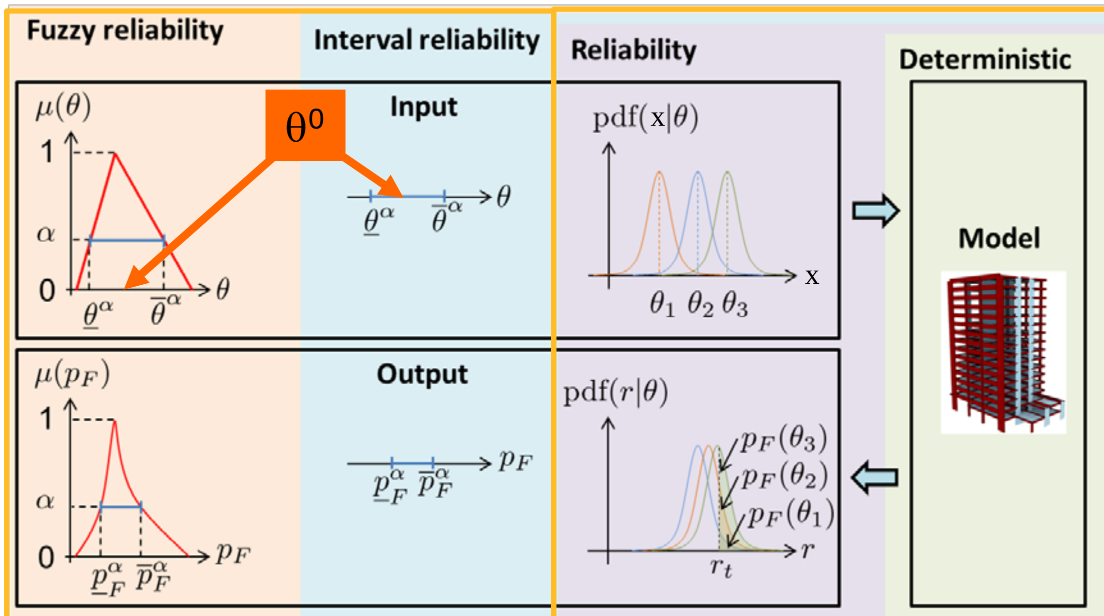
t – transformation from standard normal to physical space

n_x + 1 intervening variables $\xi(\theta)$ to capture nonlinear relation between θ and β

RELIABILITY ANALYSIS OF LARGE STRUCTURES

Parametric surrogate with intervening variables

- summary of approach



Step 1:
- apply any suitable reliability method
- estimate failure probability and sensitivity with $\theta = \theta^0$

Step 2:
- calculate parameters g_i and m_i for $\eta_i(x_i)$
- calculate h_j and specify $\xi_j(\theta)$
- construct parametric surrogate

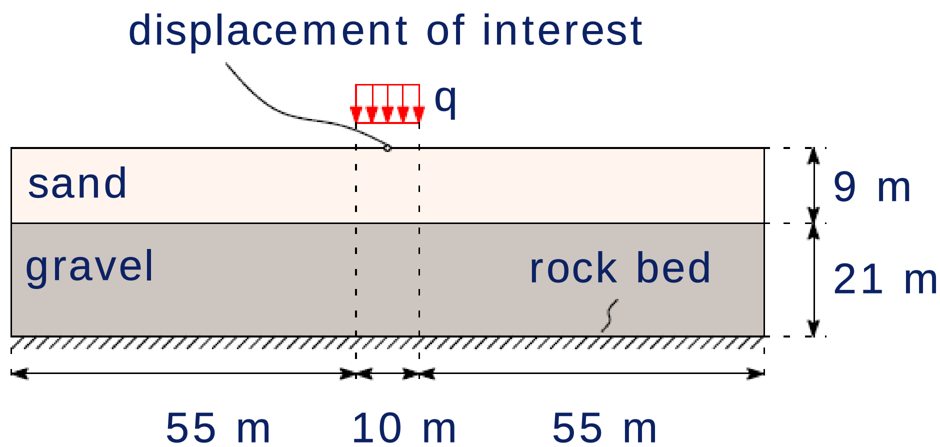
$$P_f(\theta) \approx P_f^s(\theta) = \Phi\left(h_0 + \sum_{j=1}^{n_\xi} h_j\left(\xi_j(\theta) - \xi_j(\theta^0)\right)\right)$$

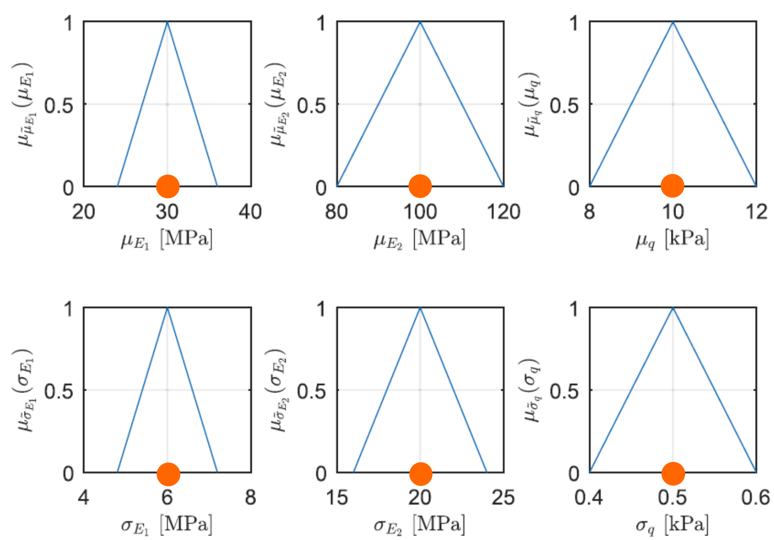
Step 3:
- estimate interval bounds or fuzzy sets for P_f using the surrogate

RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: shallow foundation

- structural model
  - » 160 quadrilateral FE in plain strain, 320 dof, linear
- random parameters
  - » load q: log-normal
  - » E_{sand}: log-normal
  - » E_{gravel}: log-normal
- fuzzy parameters
  - » expected values and standard deviations of q, E_{sand} and E_{gravel}
- P_f for exceedance of displacement of 7 cm
  - » θ^0 at fuzzy "peaks"
  - » importance sampling, N = 300

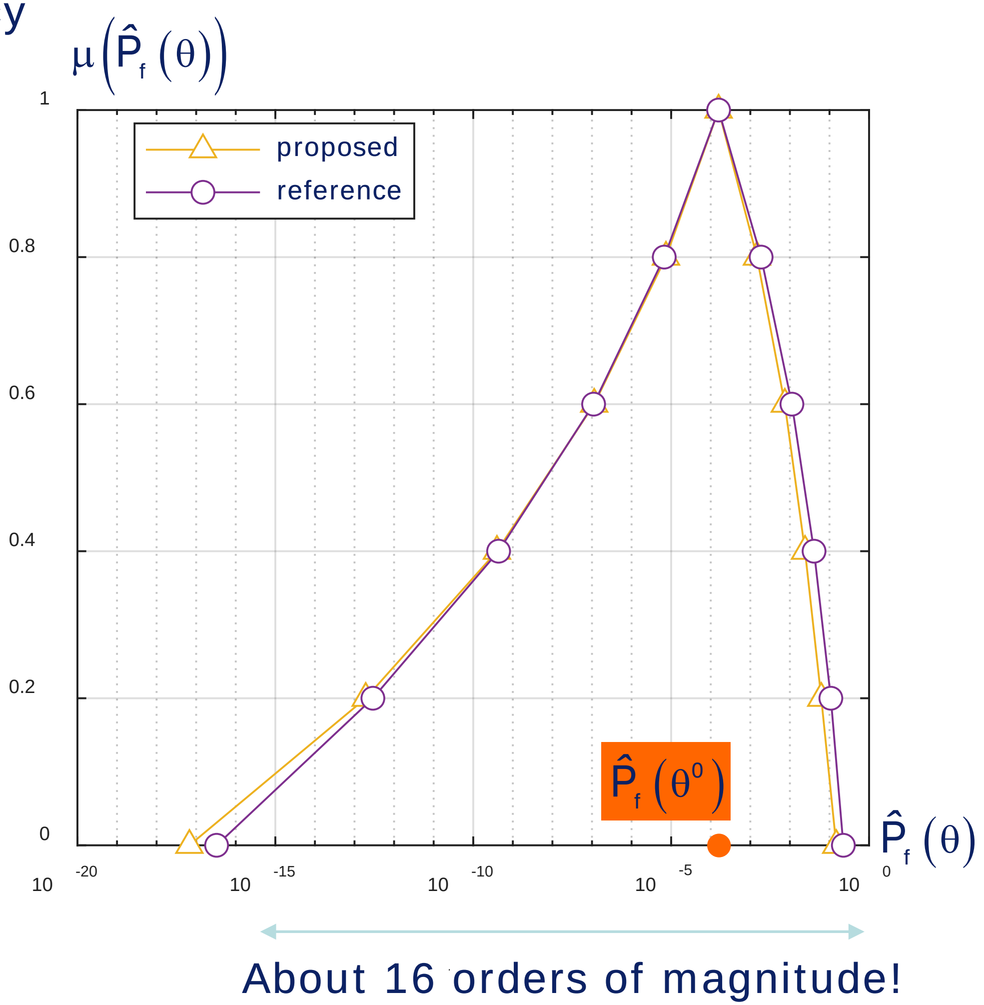




RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: shallow foundation

- results and numerical efficiency
  - » complete solution with only one reliability analysis, N=300
  - » nonlinearity of g(.) w.r.t. E_{sand} and E_{gravel}) and of P_f w.r.t. θ captured well
- application range
  - » large structures (demanding deterministic analysis)
  - » moderate nonlinearities
  - » moderate dimensionality
  - » single failure mode



RELIABILITY ANALYSIS OF LARGE STRUCTURES

Non-intrusive imprecise stochastic simulation (NISS)

- estimation of failure probability depending on parameters θ
  - » local extended MCS

$$\hat{P}_f(\theta) = \frac{1}{N} \cdot \sum_{i=1}^{N} I_F\left(x^{(i)}\right) \frac{f_X\left(x^{(i)} \middle| \theta\right)}{f_X\left(x^{(i)} \middle| \theta^*\right)}$$

    - sample $x^{(i)}$ for pre-defined $\theta^{(*)}$
    - estimate P_f in dependence on θ „extrapolating" from result at $\theta^{(*)}$
  - » global extended MCS

$$\hat{P}_f(\theta) = \frac{1}{N} \cdot \sum_{i=1}^{N} I_F\left(x^{(i)}\right) \frac{f_X\left(x^{(i)} \middle| \theta\right)}{f_X\left(x^{(i)}, \theta^{(i)}\right)}$$

    - sample $x^{(i)}$ for $\theta^{(i)}$ from auxiliary $f_\Theta(\theta)$
    - average estimate of $P_f(\theta)$ over parameter range in discretized form
- parametric surrogate for non-linear approximation w.r.t. θ
  - » high dimensional model representation (HDMR)

$$P_f^s(\theta) = \hat{P}_{f0} + \sum_{i=1}^{d} \hat{P}_{fi}\left(\theta_i\right) + \sum_{i \neq j} \hat{P}_{fij}\left(\theta_{ij}\right) + \ldots + \hat{P}_{f123\ldots d}\left(\theta_{123\ldots d}\right)$$

components reflect influence of individual parameters and parameter combinations with increasing order of interaction

Wei, P.F.; Song, J.W.; Bi, S.F.; Broggi, M.; Beer, M.; Lu, Z.Z.; Yue, Z.F. (2019):
Non-intrusive stochastic analysis with parameterized imprecise probability models: I. Performance estimation, Mechanical Systems and Signal Processing 124, 349–368.

Wei, P.F.; Song, J.W.; Bi, S.F.; Broggi, M.; Beer, M.; Lu, Z.Z.; Yue, Z.F. (2019):
Non-intrusive stochastic analysis with parameterized imprecise probability models: II. Reliability and rare events analysis, Mechanical Systems and Signal Processing 126, 227–247.

RELIABILITY ANALYSIS OF LARGE STRUCTURES

Non-intrusive imprecise stochastic simulation (NISS)

- parametric surrogate for non-linear approximation w.r.t. θ

$$P_f^s(\theta) = \hat{P}_{f0} + \sum_{i=1}^{d} \hat{P}_{fi}(\theta_i) + \sum_{i \neq j} \hat{P}_{fij}(\theta_{ij}) + \ldots + \hat{P}_{f123\ldots d}(\theta_{123\ldots d})$$

» cut-HDMR: determination of components via local extended MCS

- $\hat{P}_{f,cut,0} = \hat{P}_f(\theta^*)$
- $\hat{P}_{f,cut}(\theta_i) = \hat{P}_f(\theta_i, \theta^*_{-i}) - \hat{P}_{f,cut,0}$
- $\hat{P}_{f,cut}(\theta_{ij}) = \hat{P}_f(\theta_{ij}, \theta^*_{-ij}) - \hat{P}_{f,cut}(\theta_i) - \hat{P}_{f,cut}(\theta_j) - \hat{P}_{f,cut,0}$ etc ...

» random sampling HDMR: components via global extended MCS

- $\hat{P}_{f,RS,0} = E_{\Theta}\left[\hat{P}_f(\theta)\right]$
- $\hat{P}_{f,RS}(\theta_i) = E_{\Theta_{-i}}\left[\hat{P}_f(\theta)\right] - \hat{P}_{f,RS,0}$
- $\hat{P}_{f,RS}(\theta_{ij}) = E_{\Theta_{-ij}}\left[\hat{P}_f(\theta)\right] - \hat{P}_{f,RS}(\theta_i) - \hat{P}_{f,RS}(\theta_j) - \hat{P}_{f,RS,0}$ etc ...

facilitates consideration of set-valued structural parameters

» all components from a single sampling-based reliability analysis
» estimates of confidence bounds considering sampling uncertainty

Song, J.W.; Wei, P.F.; Valdebenito, M.; Bi, S.F.; Broggi, M.; Beer, M.; Lei, Z.X. (2019):
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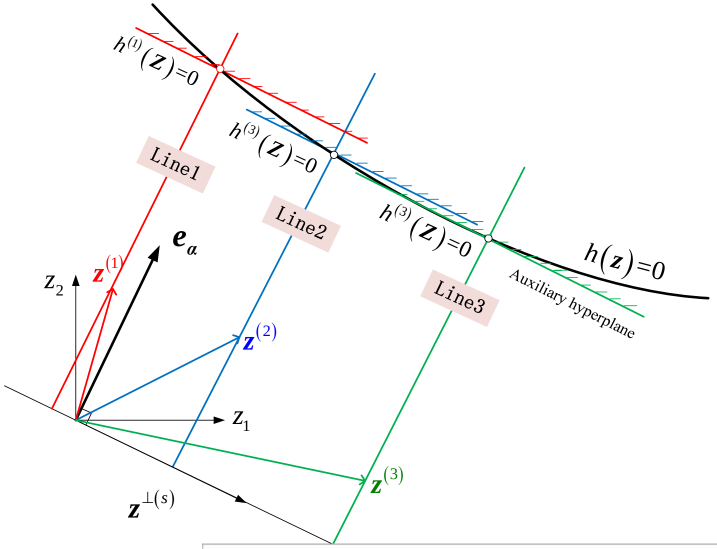
RELIABILITY ANALYSIS OF LARGE STRUCTURES

Non-intrusive imprecise stochastic simulation (NISS)

- further advancements in association with line sampling
  - » hyperplane-approximation based imprecise line sampling (local)

    approximate dependency on θ based on geometrical problem

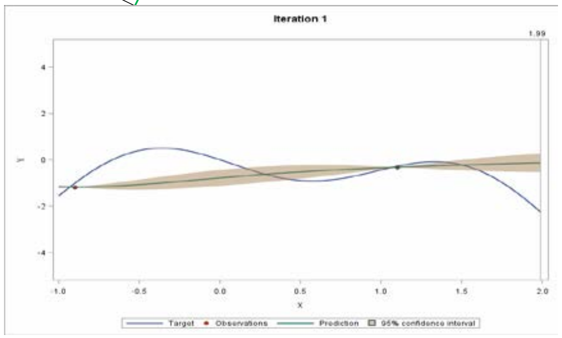


  - » weighted-integral based imprecise line sampling (local)

    approximate dependency on θ with a weighting function based on the distributions along the lines

  - » adaptive global imprecise line sampling

    Gaussian process regression model with update, distribution-based weighting function to approximate dependency on θ



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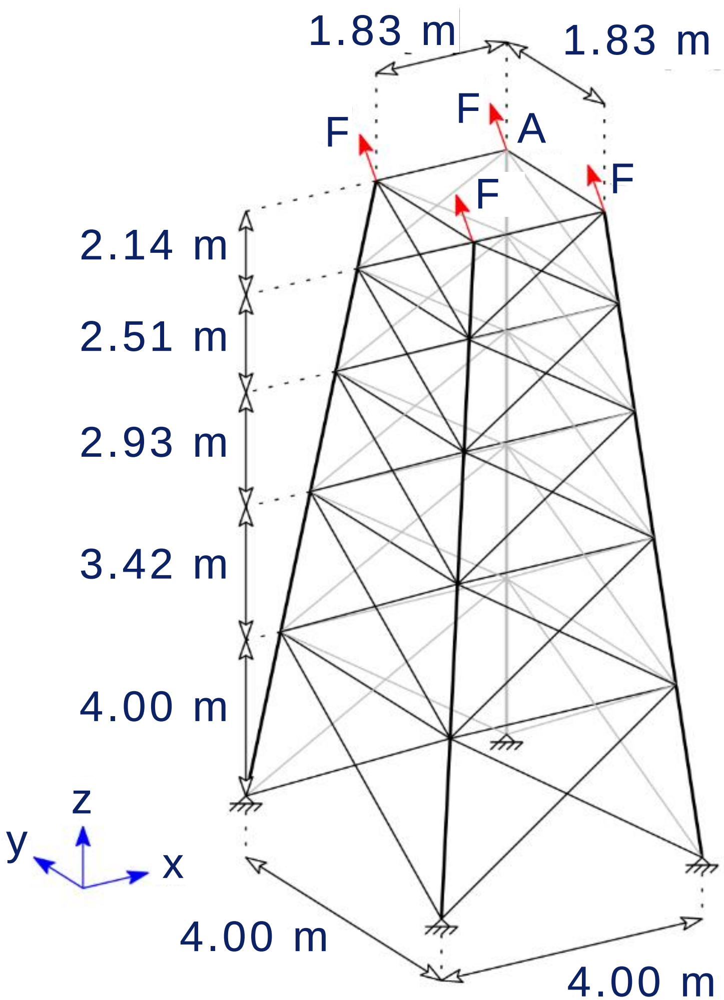
RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: transmission tower

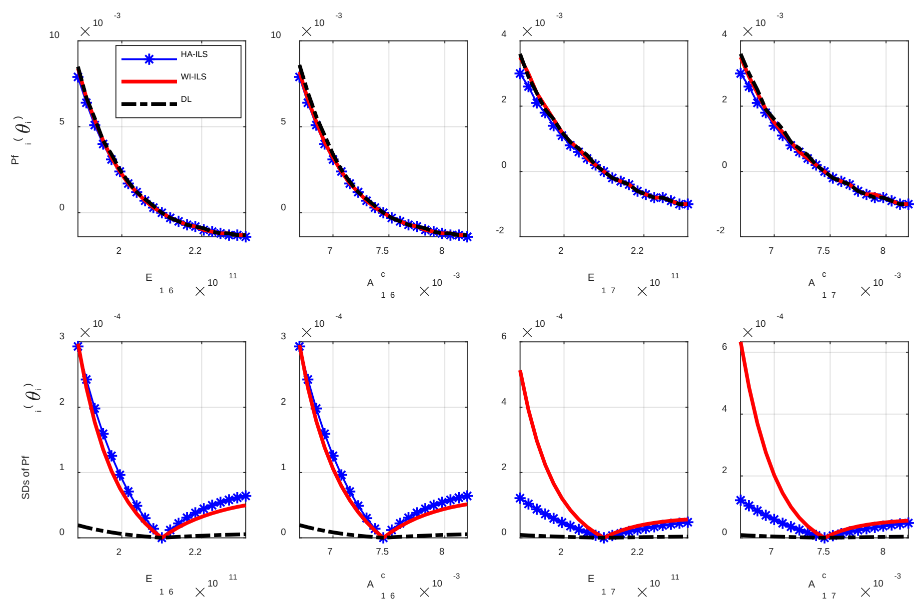
- structural model
  - » 80 bar elements, linear
  - » 4 static loads
- 160 random parameters
  - » cross-sectional area A of bars: log-normal
  - » E_{bar}: log-normal
- interval parameters
  - » expected values of A and E_{bar} for corner bars: 40 intervals
  - » cov fixed at 10%; ie interval variance with deterministic dependency
- P_f for exceedance of displacement at A of 6 cm



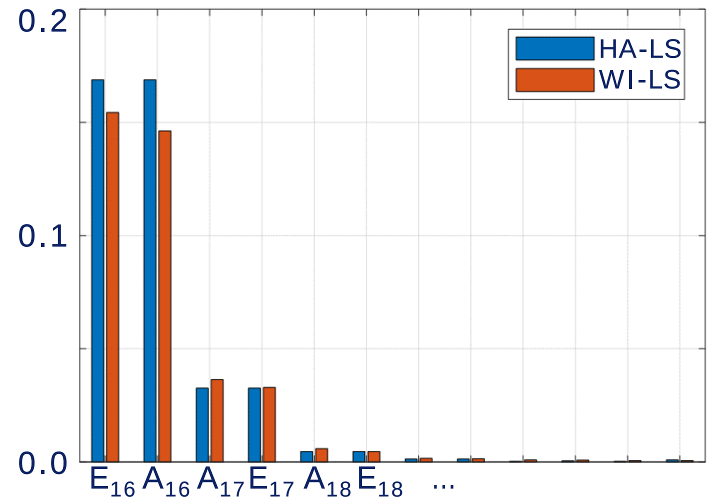
RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: transmission tower

- results and numerical efficiency
  - » local HDMR with N=1036 model calls
    (i) hyperplane approximation and
    (ii) weighted integral line sampling
  - » suitable for high dimensionality
    and moderate nonlinearities (due to LS)
- first order HDMR terms and error estimates



- sensitivity indeces for hybrid rv's

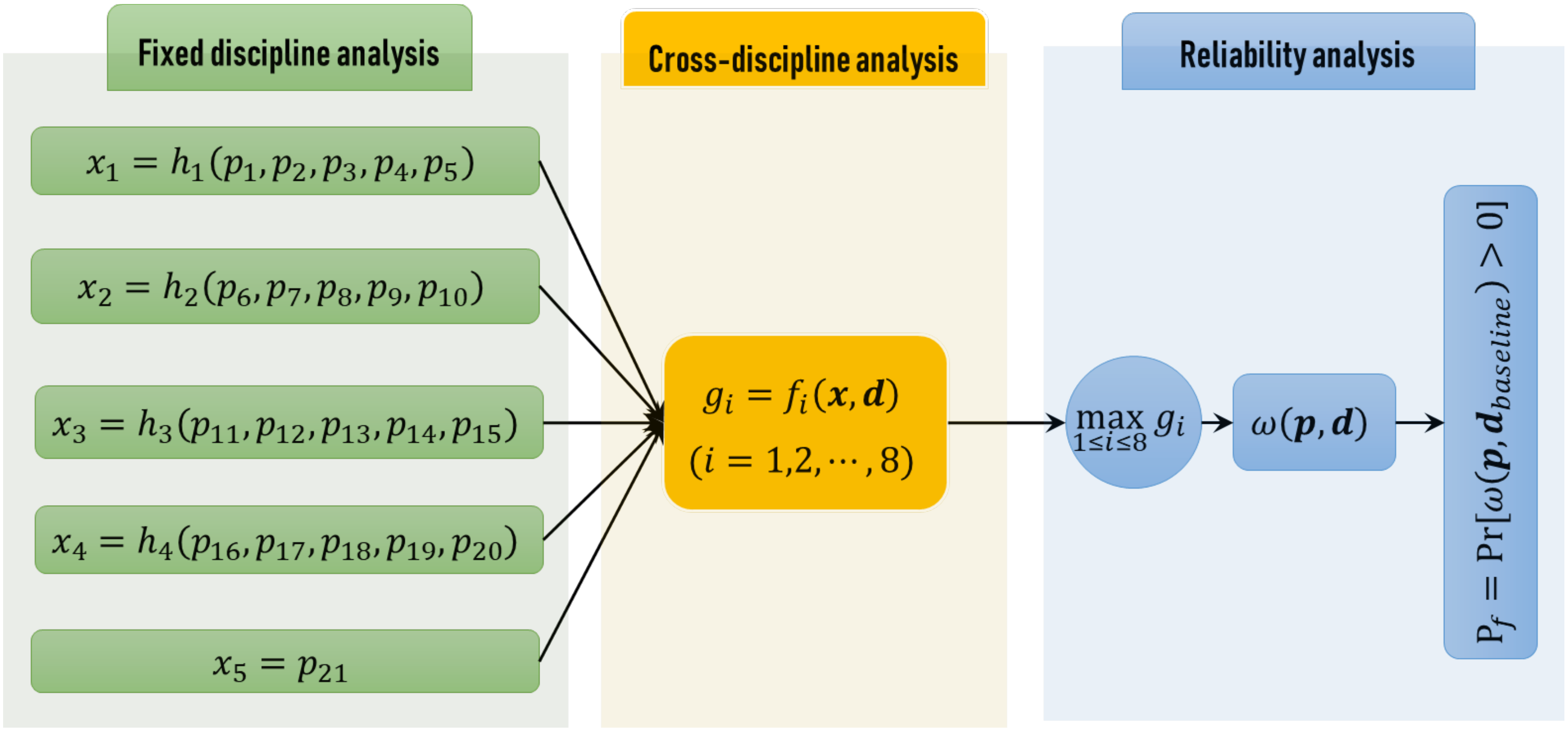


$$\hat{P}_f\left(\theta^*\right) = 1.6 \cdot 10^{-3}$$

RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: NASA Langley multidisciplinary UQ challenge problem

- problem description



RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: NASA Langley multidisciplinary UQ challenge problem

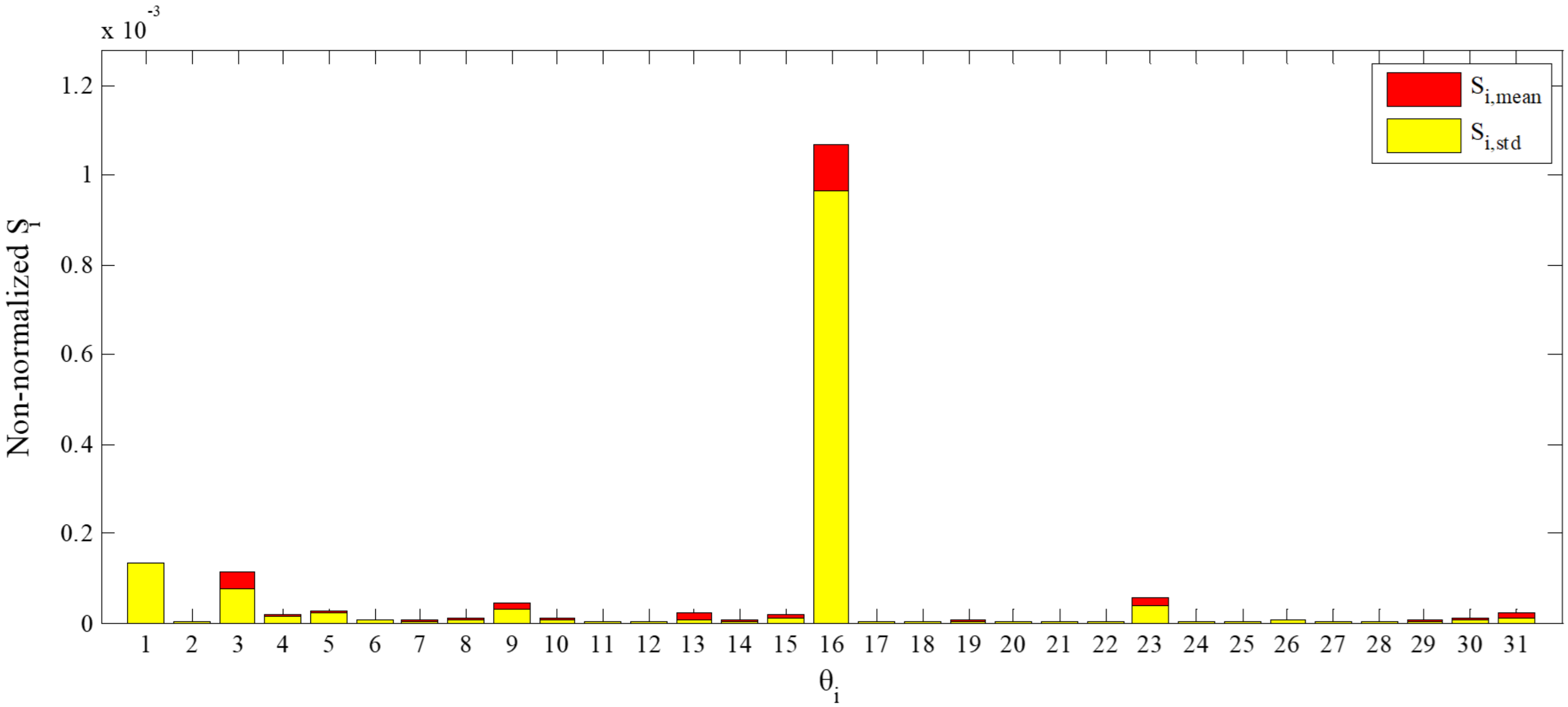
- input parameters and uncertainties

Input variables	Category	Uncertainty Characterization Models
p_1	III	Unimodal Beta, $0.6783 \leq \theta_1 = \mu_1 \leq 0.7097$, $0.0387 \leq \theta_2^2 = \sigma_1^2 \leq 0.0397$
p_2	II	Interval, $\theta_3 = p_2 \in [0.9399, 0.9902]$
p_3	I	Uniform, [0, 1]
p_4, p_5	III	Normal, $3.4493 \leq \theta_4 = \mu_4 \leq 4.5812$, $0.4190 \leq \theta_5^2 = \sigma_4^2 \leq 2.7209$, $-1.5306 \leq \theta_6 = \mu_5 \leq -0.9106$, $0.2157 \leq \theta_7^2 = \sigma_5^2 \leq 0.6914$, $-0.4370 \leq \theta_8 = \rho \leq 0.7008$
p_6	II	Interval, $\theta_9 = p_6 \in [0.2, 0.8]$
p_7	III	Beta, $0.982 \leq \theta_{10} = a \leq 3.537$, $0.619 \leq \theta_{11} = b \leq 1.080$
p_8	III	Beta, $7.450 \leq \theta_{12} = a \leq 14.093$, $4.285 \leq \theta_{13} = b \leq 7.864$
p_9	I	Uniform, [0, 1]
p_{10}	III	Beta, $1.520 \leq \theta_{14} = a \leq 4.513$, $1.536 \leq \theta_{15} = b \leq 4.750$
p_{11}	I	Uniform, [0, 1]
p_{12}	II	Interval, $\theta_{16} = p_{12} \in [0.2, 0.8]$
p_{13}	III	Beta, $0.412 \leq \theta_{17} = a \leq 0.737$, $1.000 \leq \theta_{18} = b \leq 2.068$
p_{14}	III	Beta, $0.931 \leq \theta_{19} = a \leq 2.169$, $1.000 \leq \theta_{20} = b \leq 2.407$
p_{15}	III	Beta, $5.435 \leq \theta_{21} = a \leq 7.095$, $5.287 \leq \theta_{22} = b \leq 6.945$
p_{16}	II	Interval, $\theta_{23} = p_{16} \in [0.2, 0.8]$
p_{17}	III	Beta, $1.060 \leq \theta_{24} = a \leq 1.662$, $1.000 \leq \theta_{25} = b \leq 1.488$
p_{18}	III	Beta, $1.000 \leq \theta_{26} = a \leq 4.266$, $0.553 \leq \theta_{27} = b \leq 1.000$
p_{19}	I	Uniform, [0, 1]
p_{20}	III	Beta, $7.530 \leq \theta_{28} = a \leq 13.492$, $4.711 \leq \theta_{29} = b \leq 8.148$
p_{21}	III	Beta, $0.421 \leq \theta_{30} = a \leq 1.000$, $7.772 \leq \theta_{31} = b \leq 29.621$

RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: NASA Langley multidisciplinary UQ challenge problem

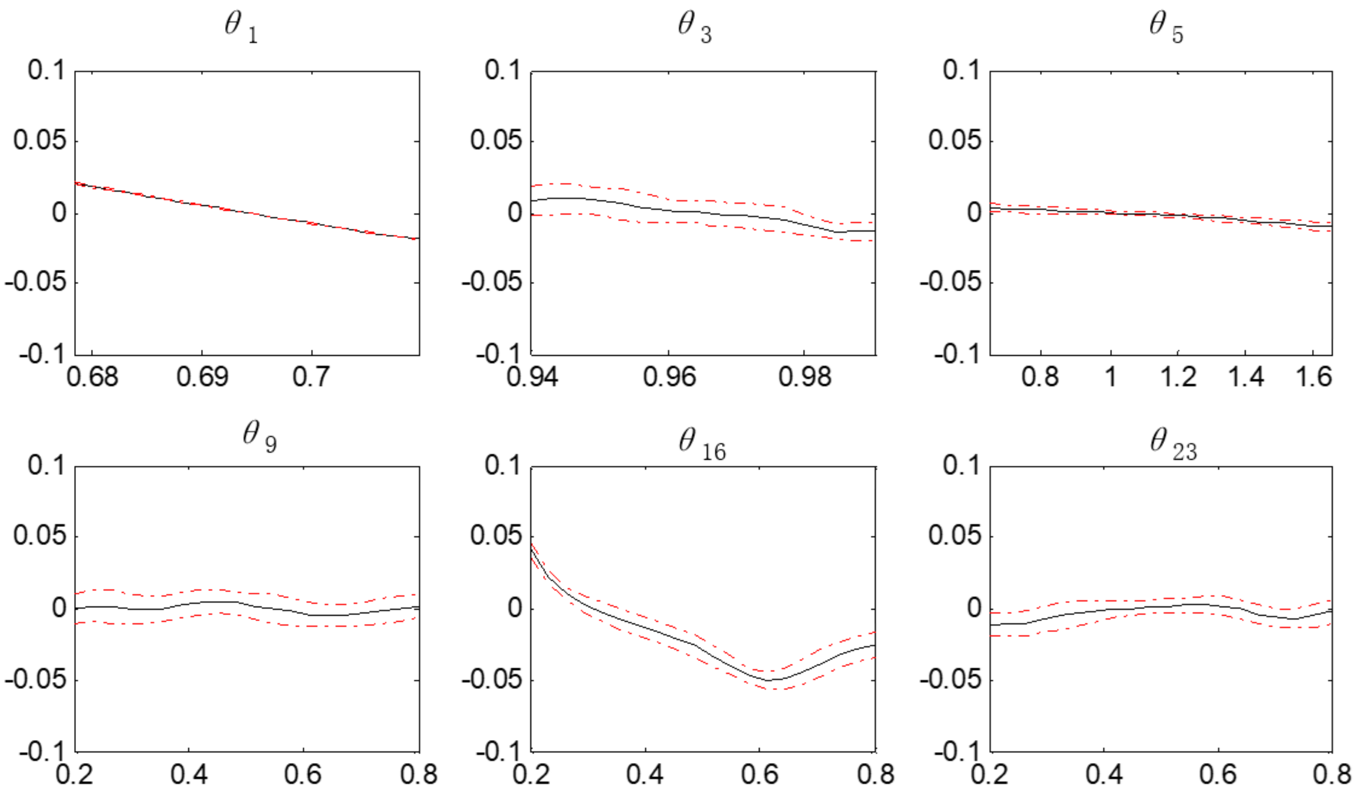
- generalized non-intrusive imprecise stochastic simulation
- first order sensitivities



RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: NASA Langley multidisciplinary UQ challenge problem

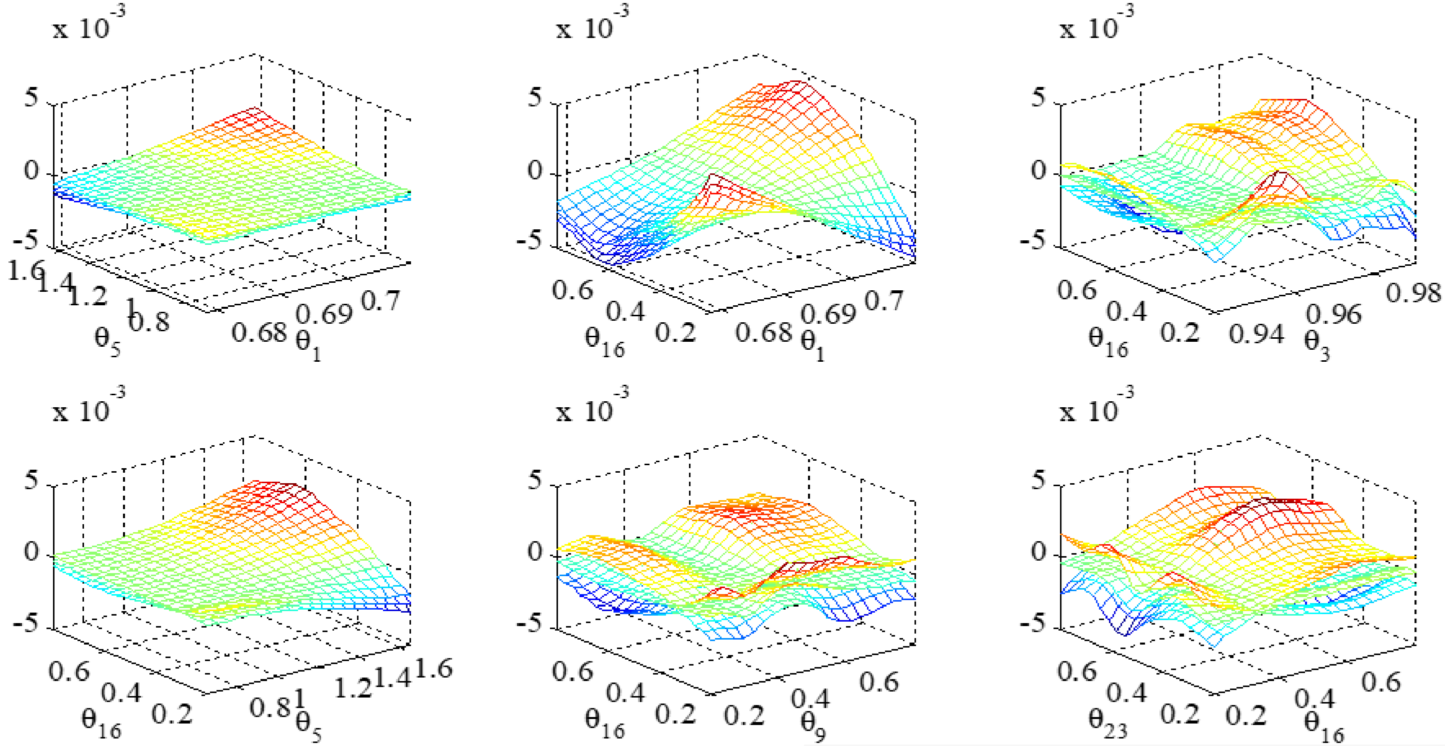
- first order global HDMR terms and 95% confidence intervals



RELIABILITY ANALYSIS OF LARGE STRUCTURES

Example: NASA Langley multidisciplinary UQ challenge problem

- second order global HDMR terms and 95% confidence intervals



Failure Probability Bounds:

NISS: [0.1221, 0.3121], N=5·10^4

IMCS: [0.0550, 0.3370], N=10^7

- moderate to high dimensionality, strong nonlinearities

RESUMÉ

Efficient numerical methods for imprecise probabilities

- compatible with stochastic approaches and techniques
- applicable to nonlinear and high-dimensional problems
- quantitative set-theoretical consideration of epistemic uncertainty
- comprehensive reflection of imprecision in the computational results; bounds on probabilities
- identification of sensitivities wrt. imprecision of structural and stochastic models

➡ realistic models

➡ efficient numerical analysis

➡ UQ for industry-sized structures and systems

➡ improved design, performance and reliability

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