Engineering and Imprecise Probability

What's going on?

Alice Cicirello (TU Delft), Matthias Faes (TU Dortmund), Edoardo Patelli (Strathclyde University)

Who we are

Alice Cicirello

Associate Prof & Head of the Mechanics and Physics of Structure Section at **TU Delft**.

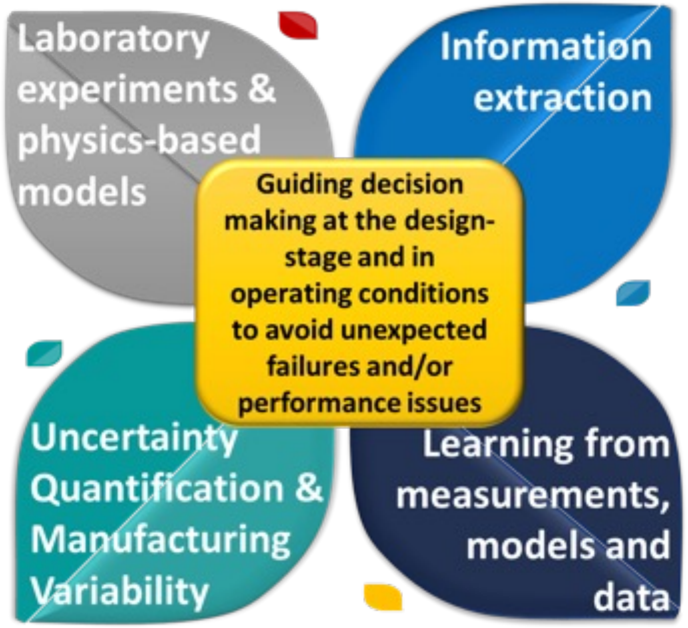
Department of Engineering Science, University of Oxford

The University Of Sheffield.

Alexander von Humboldt Stiftung/Foundation

Data, Vibration, and Uncertainty Group

https://sites.google.com/view/dvugroup/home



Matthias Faes

- Professor at TU Dortmund University
- Head of the Chair for Reliability Engineering
  Ph.D. in Mechanical Engineering from KU Leuven in 2017
- Research interests:
  - numerical methods for uncertainty quantification
  - reliability based design optimization
  - uncertainties in structural dynamics
  - non-deterministic model updating
  - many more ☺

technische universität dortmund

Chair for Reliability Engineering

Matthias Faes

The work in this presentation is a collaboration with Marcos Valdebenito, Xiukai Yuan, Chao Dang, Pengfei Wei, Marc Fina, David Moens and Michael Beer (among many others).

Edoardo Patelli

Professor in Risk and Uncertainty Quantification

Head of **Centre for Intelligent infrastructure**,

Department of Civil and Environmental Engineering

University of Strathclyde

- Assessment of safety critical systems
- Resilient engineering
- Uncertainty quantification and advanced simulation techniques
- Machine learning and artificial intelligence
- Human reliability and interaction autonomous systems

University of Strathclyde Glasgow

Edoardo Patelli

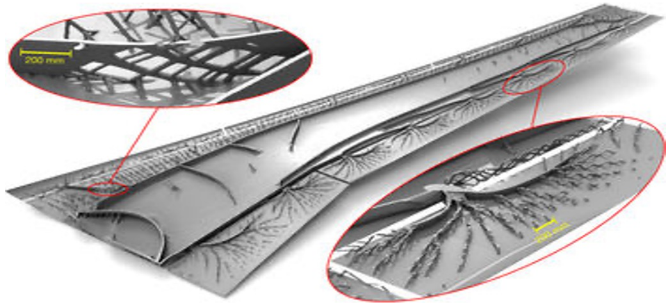
Most of the work presented is in collaboration with Caroline Morais, Ander Gray, Enrique Miralles-Dolz, Marco de Angelis, Adolphus Lye, and Scott Ferson



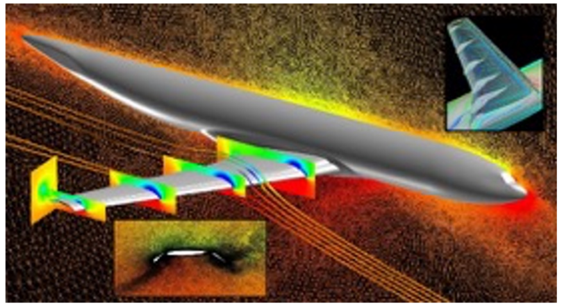



Introduction

Numerical methods in engineering

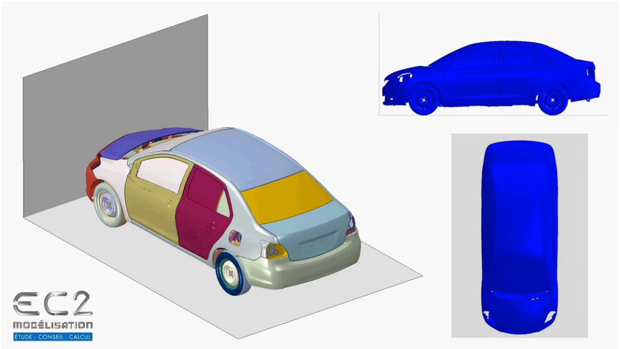


Topology optimised Airplane wing
(Nature)

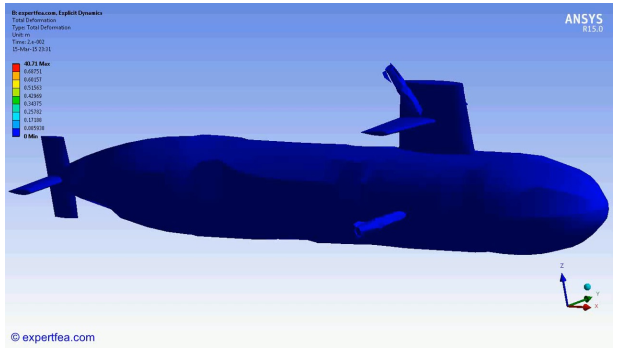


Airflow around Boeing 737 body
(NASA / Boeing)

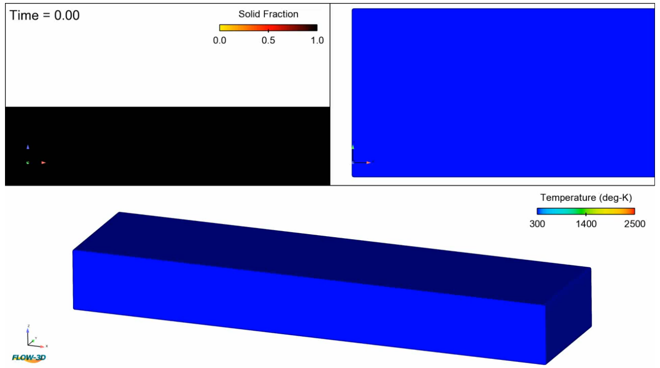
Wendelstein 7-X fusion reactor
(Max Planck Institute)



Car crash simulation
(Toyota Yaris)



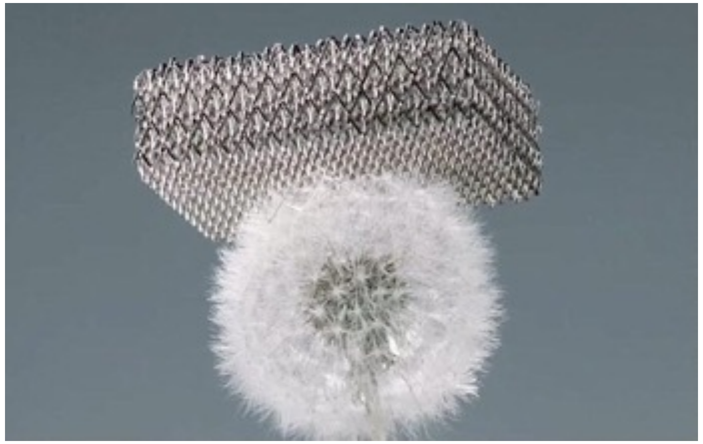
Dual torpedo impact
(Ansys)

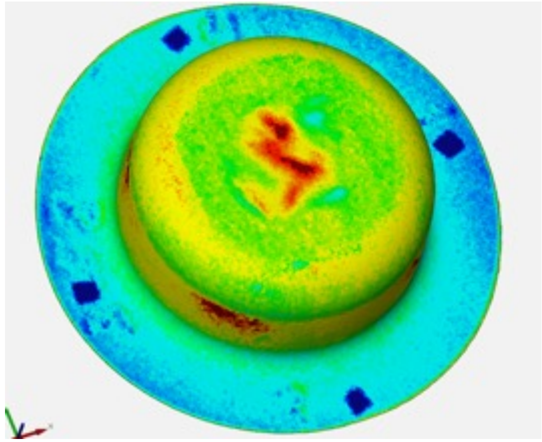


Laser Metal Deposition
(FlowScience)

Why do we want to have these models?

- Numerical experimentation is usually cheaper than experimental work
  - Optimization of complex products and systems
  - Improve understanding of complicated processes
  - Ensure mechanical reliability of products
  - Risk analysis of critical infrastructure

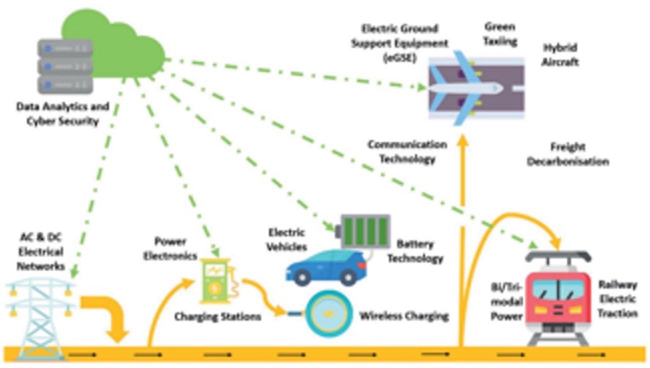


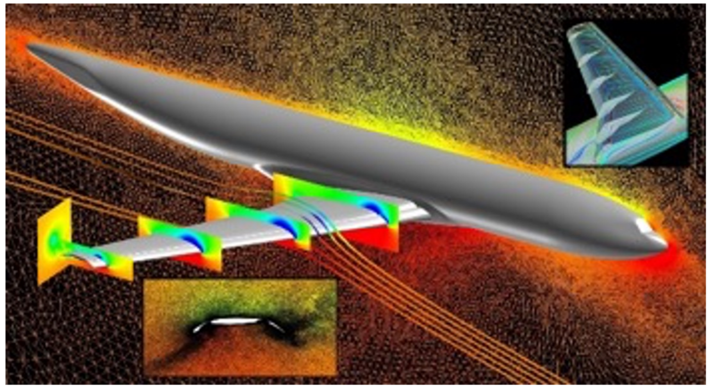


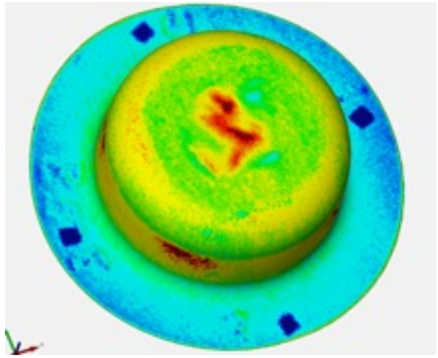
> *With great power comes great responsibility.*
>
> *- Voltaire,* 1832

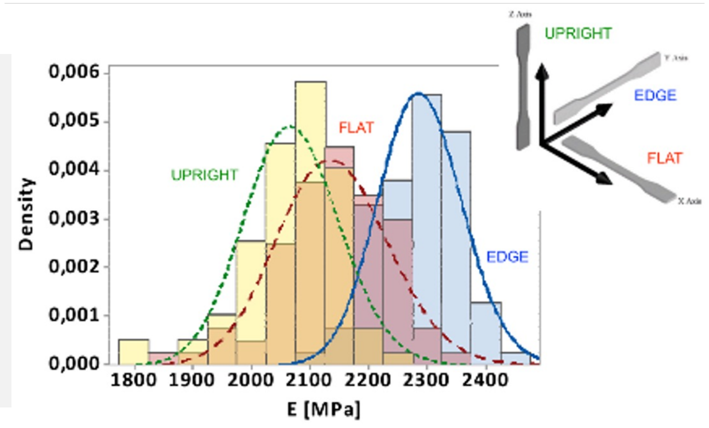
However…

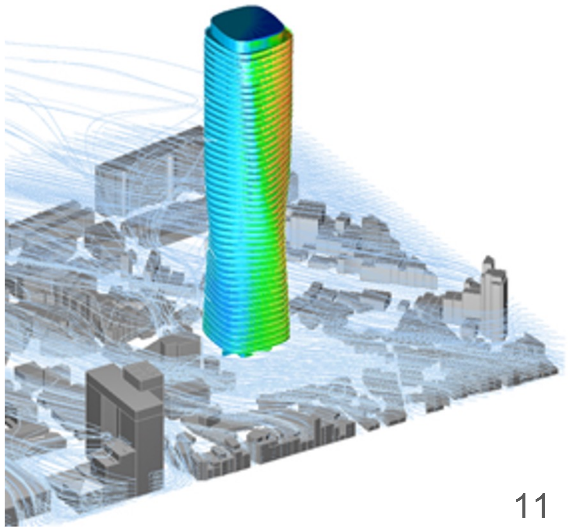
- complex physics / systems require complicated simulation models
- these models require many inputs and settings to be defined:
  - macro and micro scale inter- and intra-variability,
  - loading is often insufficiently known or appears as inherently variable,
  - underlying physics are often approximated,
  - humans build and interpret the models and their results



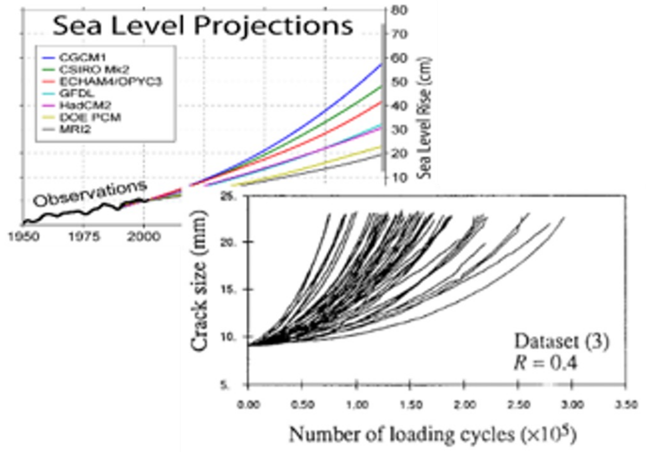








Requirements and challenges

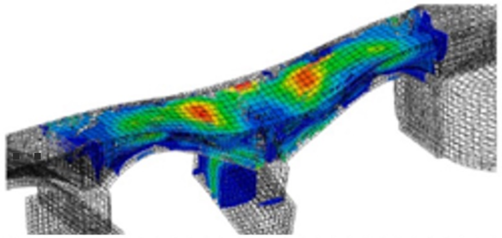


Data to identify threats and system performance but ….

data is incomplete/scarce, noise, missing or ambiguous, tampered

Models to predict system behavior but …

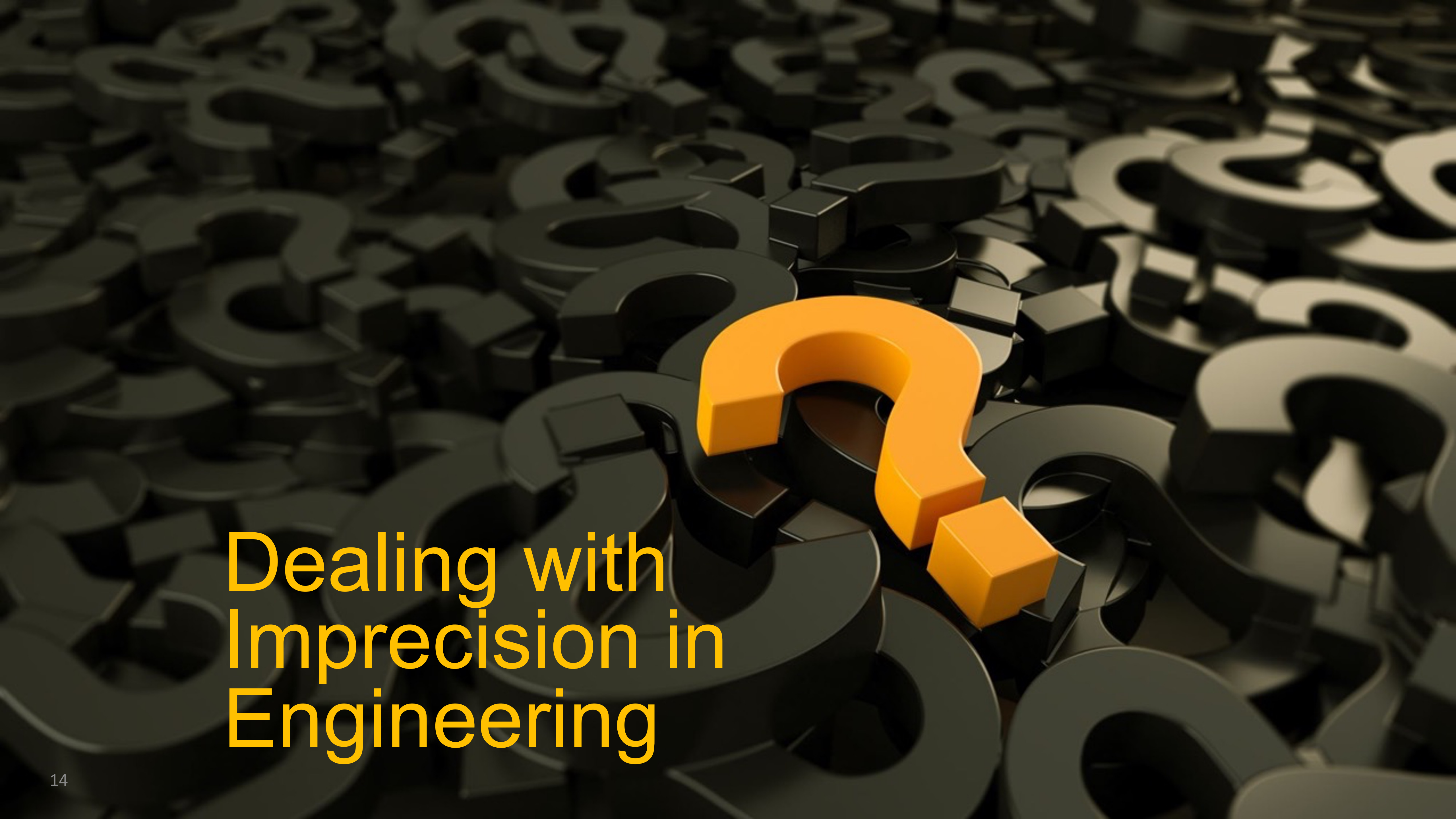
models are approximation of reality and/or computational expensive



Decision tools for optimal design/maintenance/recovery but…

ignoring uncertainty, unjustified assumptions

Data analytics ← → Physical modelling



Dealing with Imprecision in Engineering

The data problem

Data overflow and data shadowing

Data availability is expect to increase

Cheap sensors, tracking and logs, surveillance

Are the data relevant?

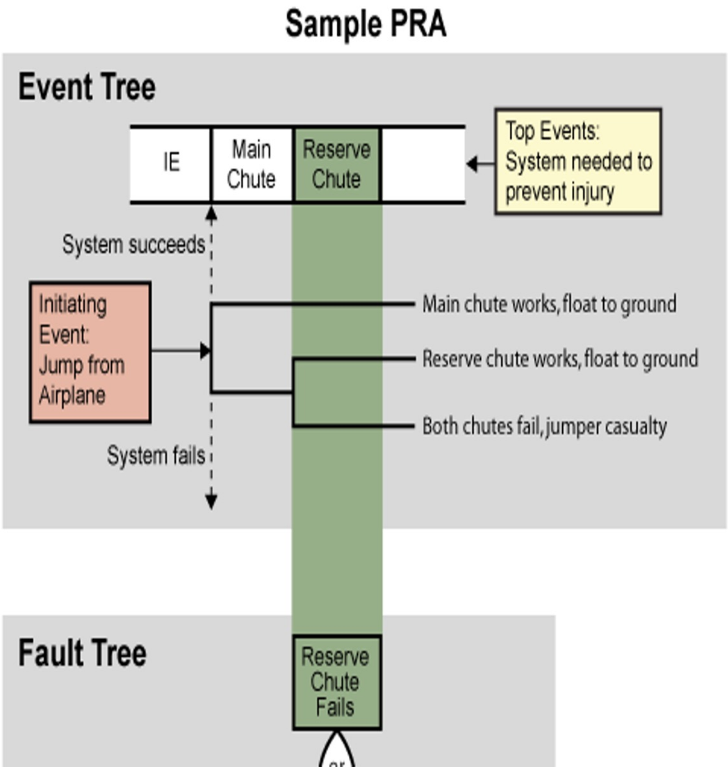
Are we measuring the right thing?

Machine learning and AI should support uncertainty characterisation

Data robustness and trustfulness

Data tampering and malicious injection of false data

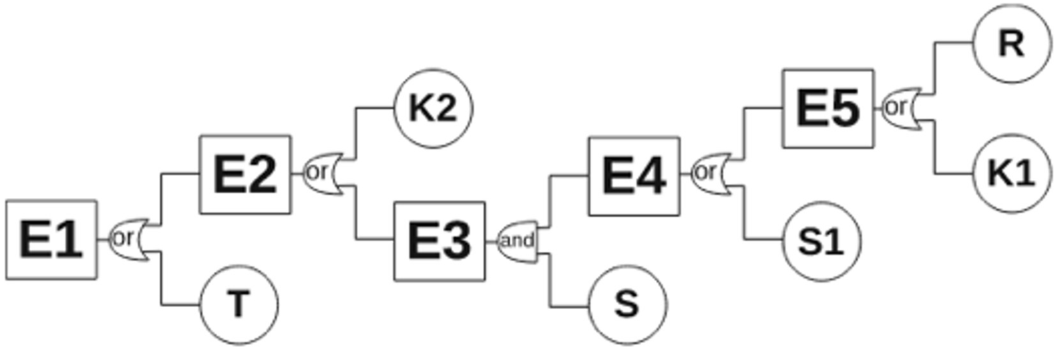
Nuclear power plant

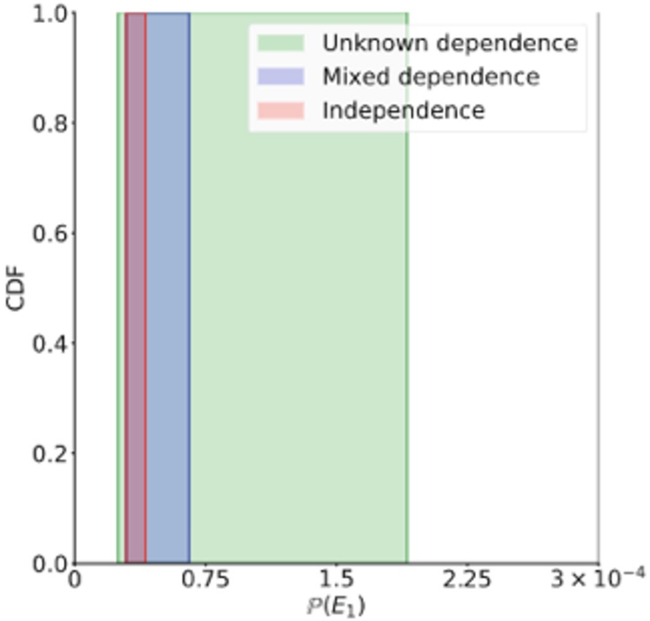


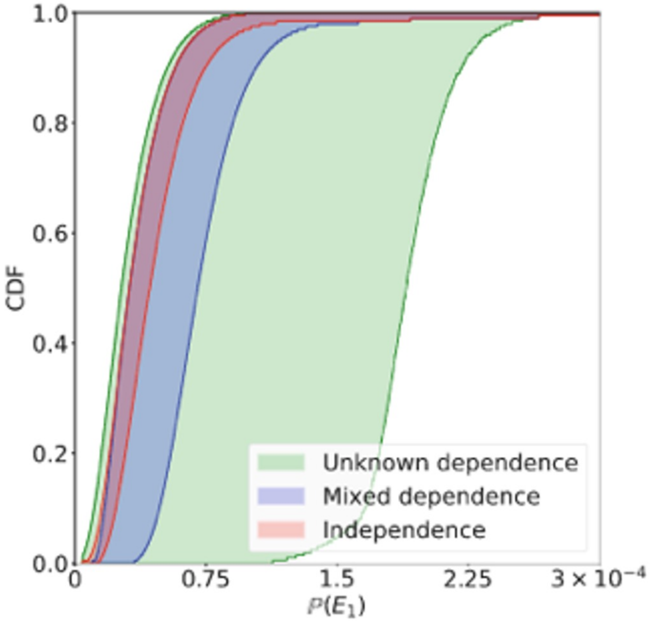
Methodologies, licensing, and regulation of "matured" engineering systems are well established

Risk analysis

Independence (of events) is often assumed it can have dramatic consequences.





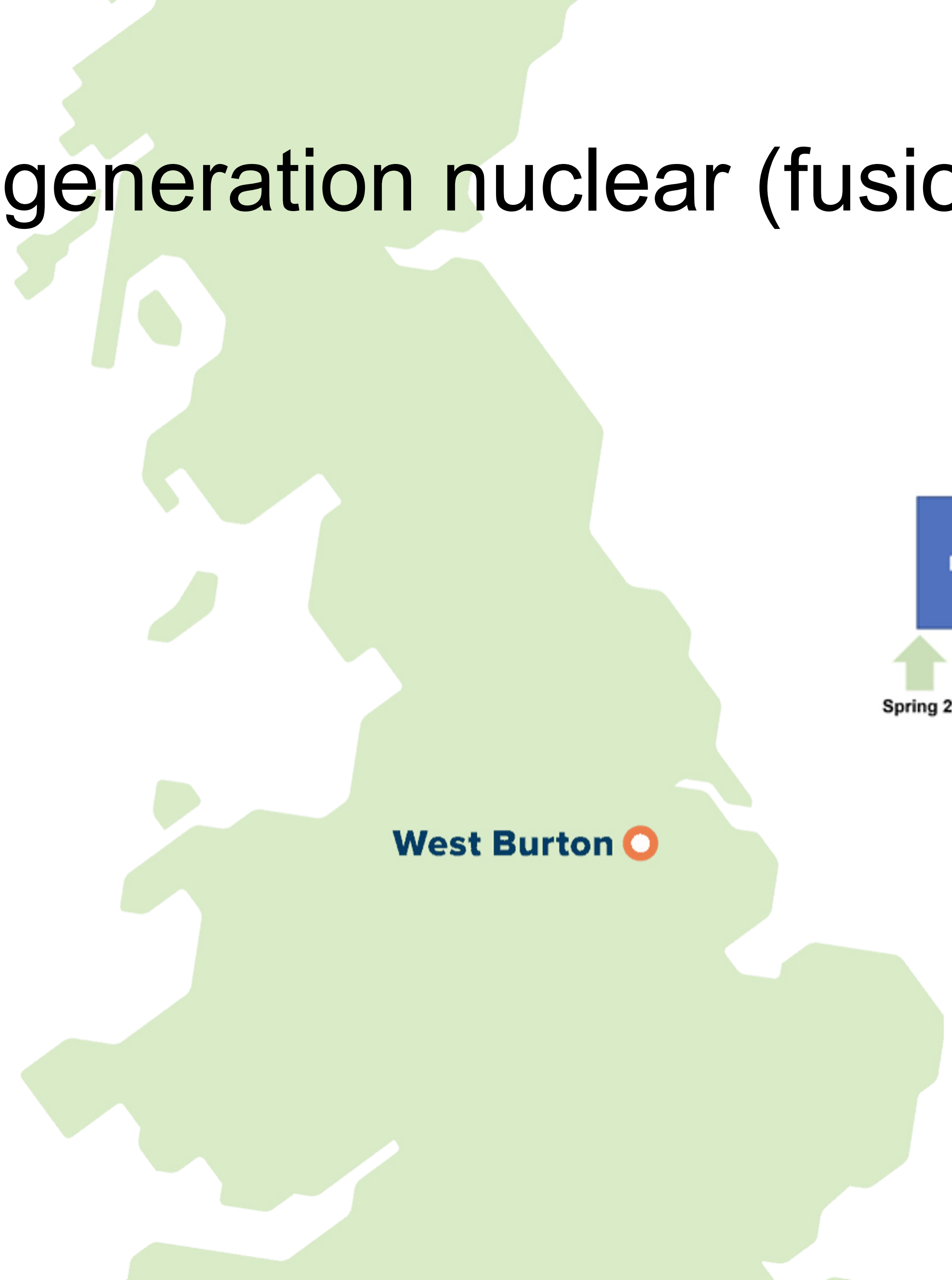


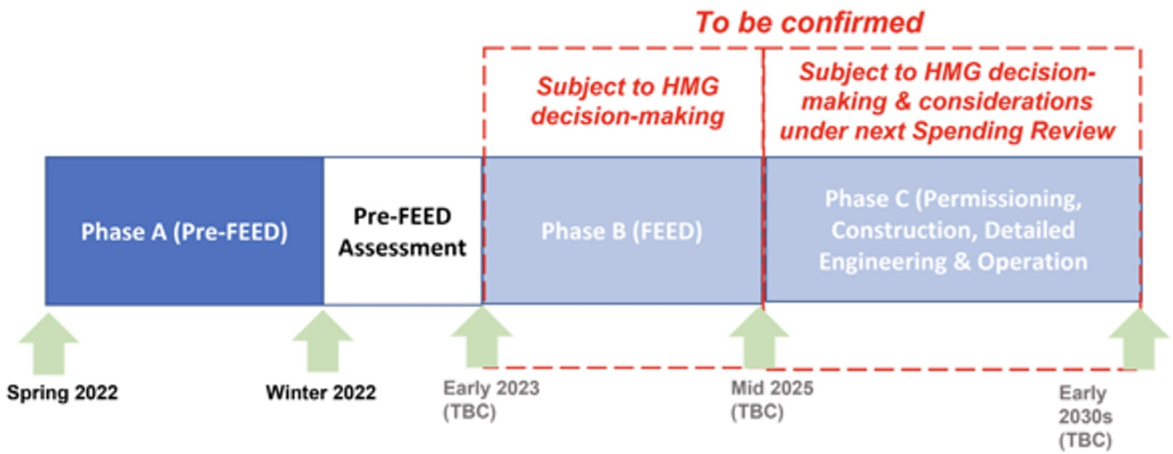
Component	Interval scenario	P-box scenario
T	$[4.5 \times 10^{-6}, 5.5 \times 10^{-6}]$	$[4.5 \times 10^{-6}, 5.5 \times 10^{-6}]$
K2	$[2.5 \times 10^{-5}, 3.5 \times 10^{-5}]$	$KN(3, 10^5)$
S	$[0.5 \times 10^{-4}, 1.5 \times 10^{-4}]$	$[0.5 \times 10^{-4}, 1.5 \times 10^{-4}]$
K1	$[2.5 \times 10^{-5}, 3.5 \times 10^{-5}]$	$KN(3, 10^5)$
R	$[0.5 \times 10^{-4}, 1.5 \times 10^{-4}]$	$KN(1, 10^4)$
S1	$[2.5 \times 10^{-5}, 3.5 \times 10^{-5}]$	$[2.5 \times 10^{-5}, 3.5 \times 10^{-5}]$

Event	Independence	Mixed dependence	Unknown dependence
E_1	$\rho = 0$	$\rho = 0$	$\rho = [-1, 1]$
E_2	$\rho = 0$	$\rho = [-1, 1]$	$\rho = [-1, 1]$
E_3	$\rho = 0$	$\rho = 0.15$	$\rho = [-1, 1]$
E_4	$\rho = 0$	$\rho = [-0.2, 0.2]$	$\rho = [-1, 1]$
E_5	$\rho = 0$	$\rho = 1$	$\rho = [-1, 1]$

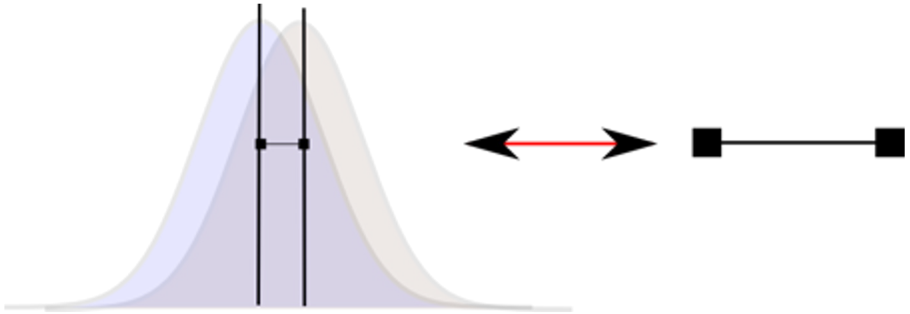
Miralles-Dolz, E., A.Gray, E.Patelli, S.Ferson. 'Correlated Boolean Operators for Uncertainty Logic'. In Information Processing and Management of Uncertainty in Knowledge-Based Systems, 798–811. Communications in Computer and Information Science. Cham: Springer International Publishing, 2022. https://doi.org/10.1007/978-3-031-08971-8_64.

Next generation nuclear (fusion) plant





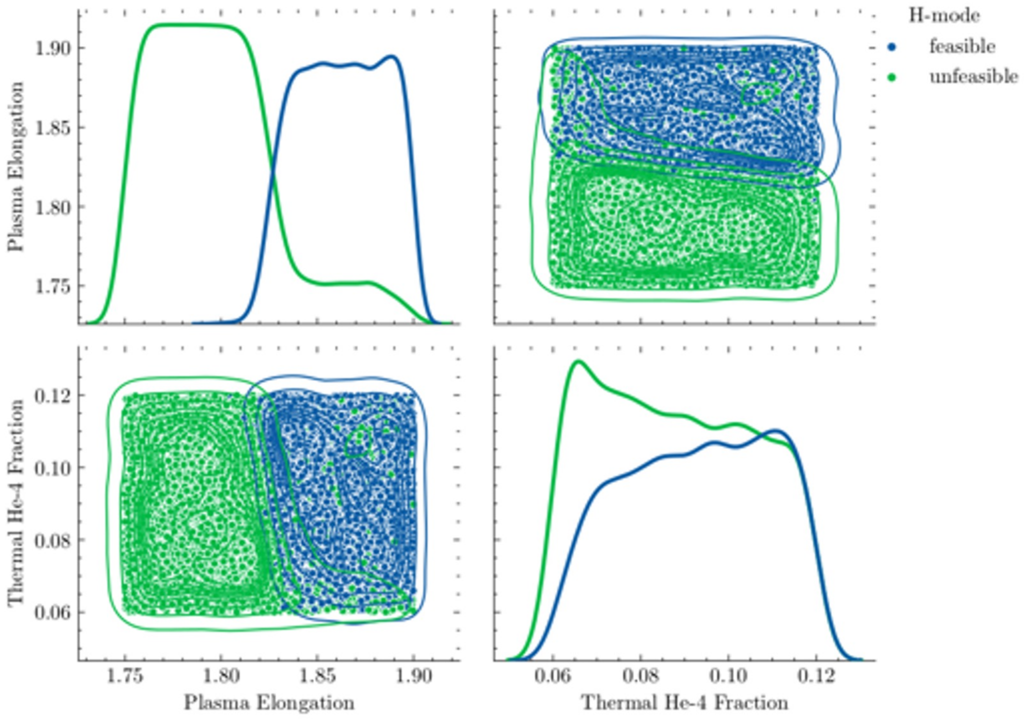
HTGR demonstration programme



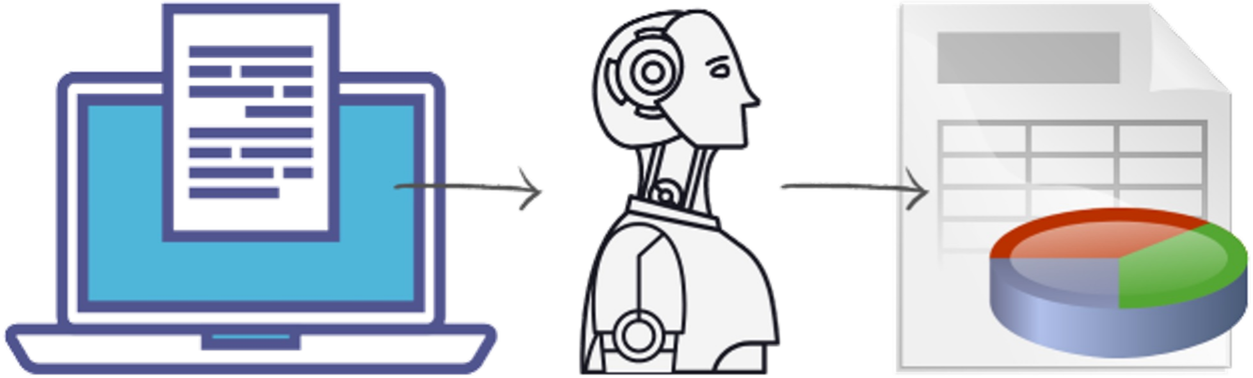
Design of DEMO Nuclear fusion reactor

It is essential considered the possible variability of the design parameters

Parameter	Lower bound	Upper bound	Baseline
H-factor	1.0	1.3	1.1
Divertor Limit (MWT/m)	8.7	9.5	9.2
Core Radius	0.6	0.8	0.75
W Impurity	10^{-5}	10^{-4}	50^{-5}
Plasma Elongation	1.75	1.90	1.85
Thermal He-4 fraction	0.06	0.12	0.069



Miralles-Dolz, E., A.Pearce, J.Morris, E.Patelli. 'Toward DEMO Power Plant Concept Selection Under Epistemic Uncertainty'. *IEEE Transactions on Plasma Science*, 2022, 1–6. https://doi.org/10.1109/TPS.2022.3180233.

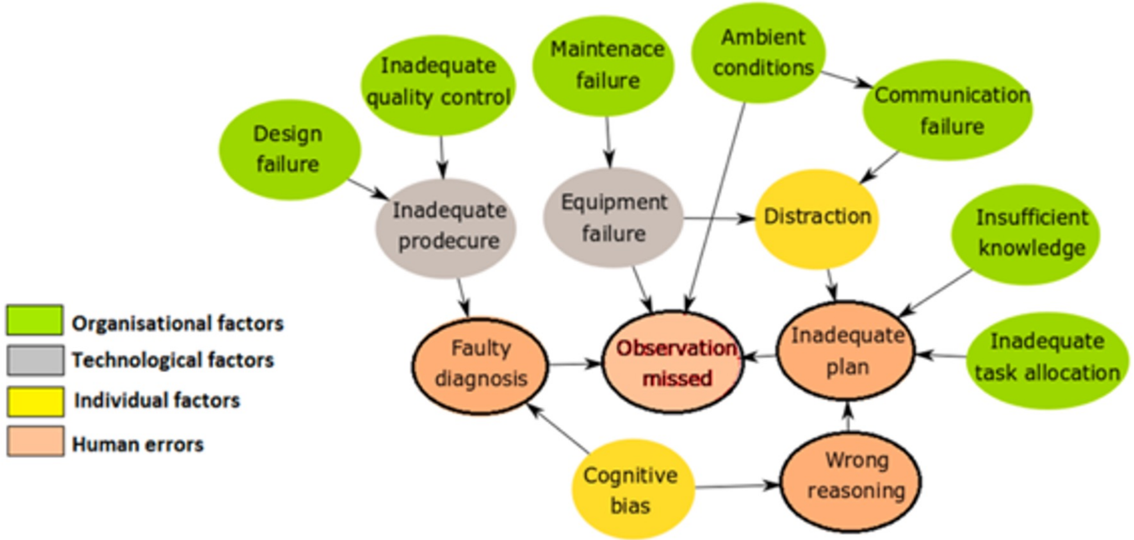


VIRTUAL HUMAN FACTORS CLASSIFIER

An artificiall intelligence tool to classify an accident report into a human reliability classification scheme (such as human errors, organizational and technological factors)

Assessment of Human reliability

- Human is the ultimate barrier for safety also a source of uncertainty and failure
- Need to quantify the performance of operators and personnel
- Often modelled by Bayesian Networks



Morais, C., Moura, R., Beer, M., Patelli, E., 2020. Analysis and Estimation of Human Errors From Major Accident Investigation Reports. ASCE-ASME J Risk and Uncert in Engrg Sys Part B Mech Engrg 6, 011014. https://doi.org/10.1115/1.4044796

Assessment of Human reliability

- Learn from real data - Multi-attribute Technological Accidents Dataset (MATA-D)
- Machine-learning classification for new accident report (continuously updated)
- Natural Language Processor for text analytics

Morais, Caroline, Ka Lai Yung, Karl Johnson, Raphael Moura, Michael Beer, and Edoardo Patelli. 'Identification of Human Errors and Influencing Factors: A Machine Learning Approach'. *Safety Science* 146C (2022): 105528. https://doi.org/10.1016/j.ssci.2021.105528.

The data problem I (missing data)

Some events are never been observed (and this does not imply a null probability of the evens)

- No observations is different than 0 (null) probability

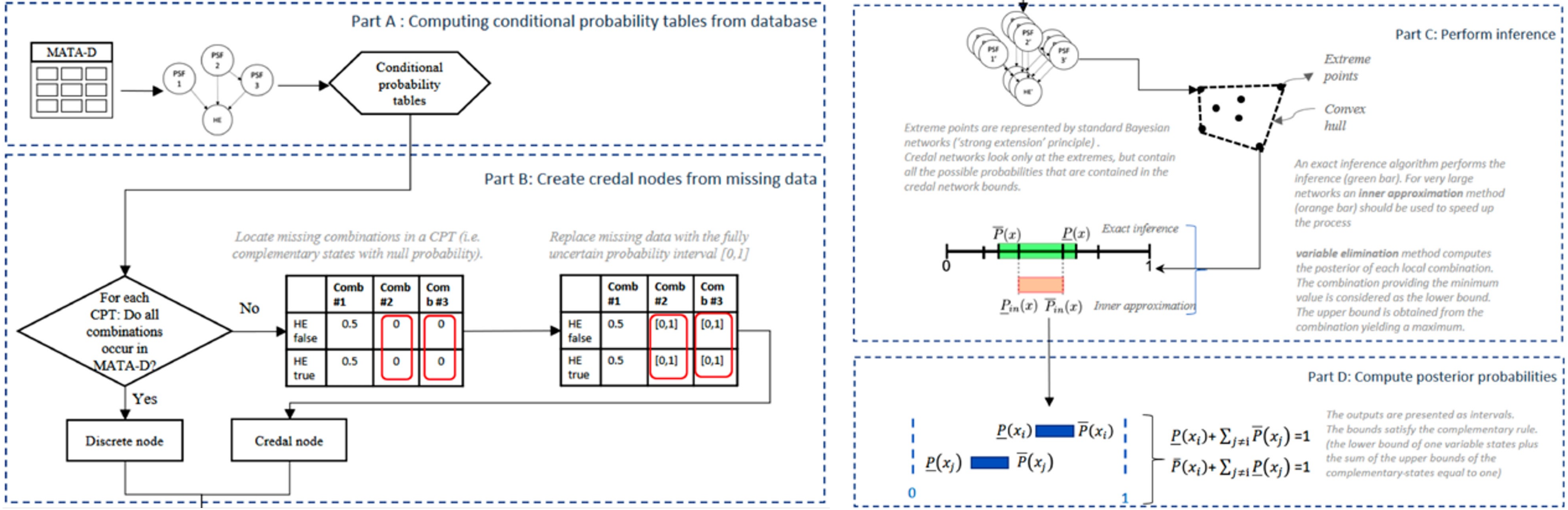
	Design failure	Inadequate procedure	Observation missed
Accident 1	1	0	0
Accident 2	1	1	1
...	0	1	0
...	1	0	0
...	1	1	1
...	0	0	1
Accident 238	1	1	1
Frequency	66.0%	44.1%	15.5%

Approaches in Human Reliability Analysis

- Assign equal probability for both states
  Standard approach in Bayesian Network software (Genie)
- *Linear interpolation* algorithms and Cain calculator
  approximating CPD anchors with functions, interpolating amongst available CPDs to obtain full set of approximating functions, and discretizing them back to obtain the full set of CPTs
  The Cain method directly exploits monotonicity to determine the proportion of change in the child states probabilities from parent nodes and missing relationships in CPTs
- *Expert elicitation*
- *Noisy-OR (and extensions)*
- *'not applicable'* state
- *Artificial data*
  implies the generation of data with known properties by an algorithm rather than expert opinion.

Credal networks

Defining the intervals to replace missing data combinations



	Comb #1	Comb #2	Com b #3
HE false	0.5	0	0
HE true	0.5	0	0

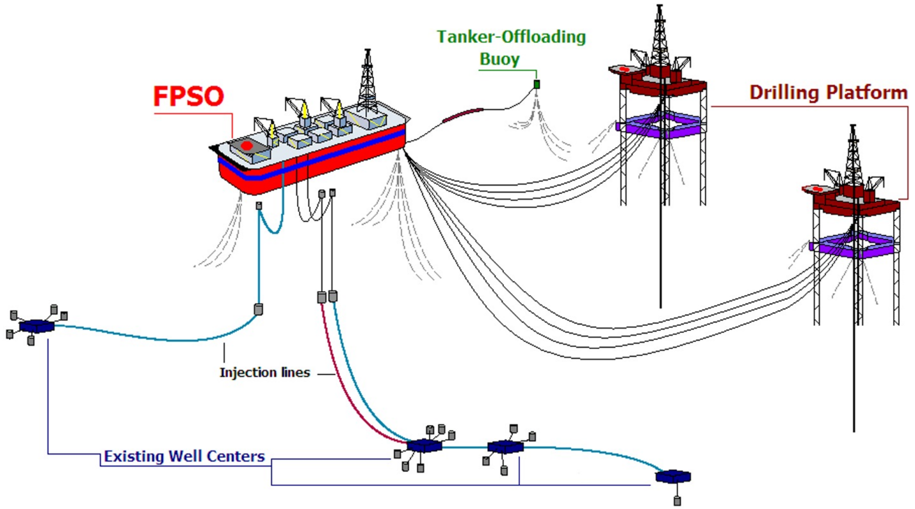
	Comb #1	Comb #2	Comb #3
HE false	0.5	[0,1]	[0,1]
HE true	0.5	[0,1]	[0,1]

Case study

Quantify the human reliability of operator during the storage tank depressurisation on static offshore oil & gas installations

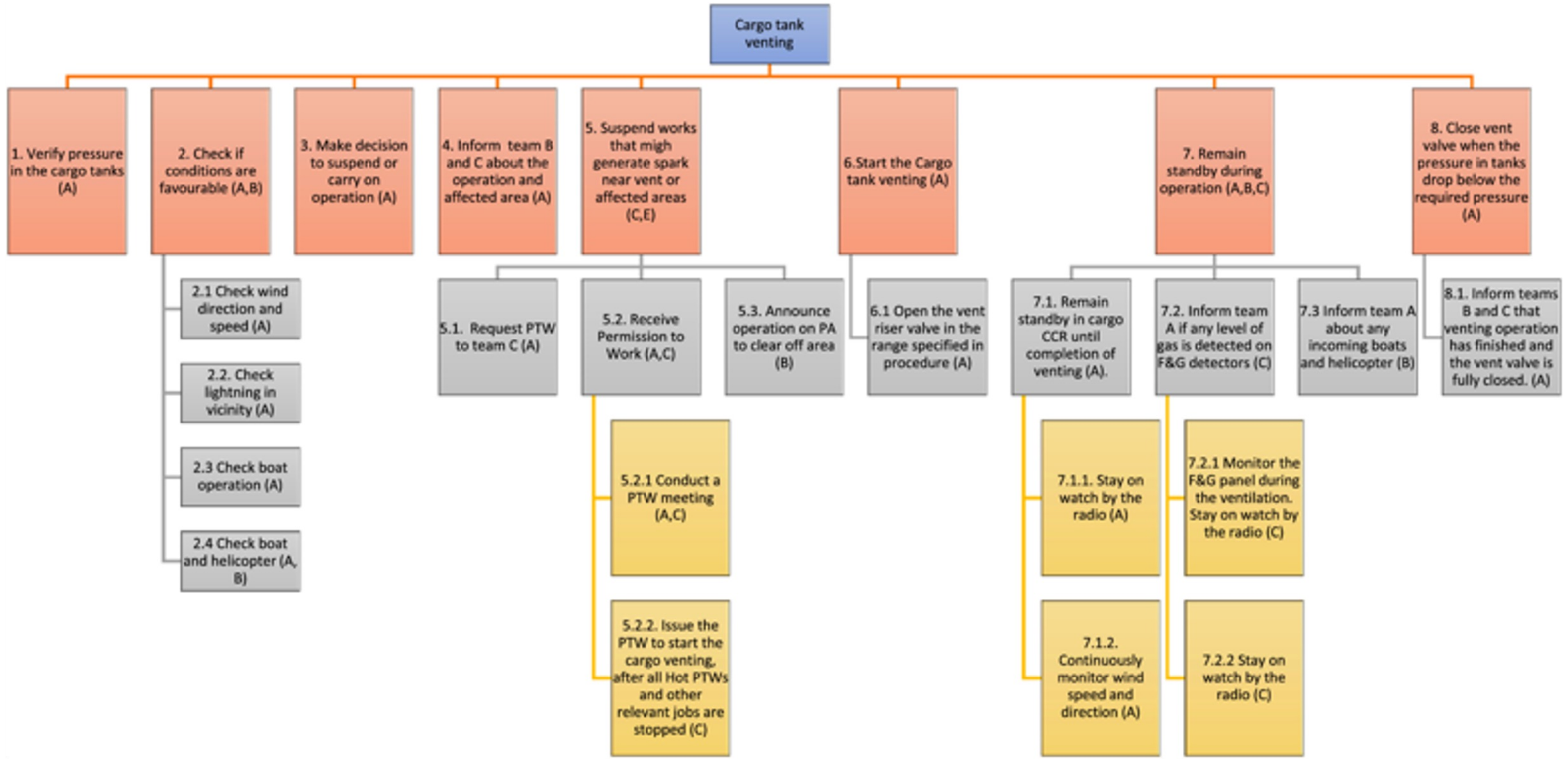
Hazards: under certain wind conditions the vapours released might reach a source of ignition (e.g., other equipment, operations and maintenance works) with the potential to cause fire, explosion or financial loss due to emergency production shutdown

The **operators** are the main barriers to prevent an incident event, with little or no support from automatic systems/technology



Identified Credal Network

Nodes identified with critical task analysis



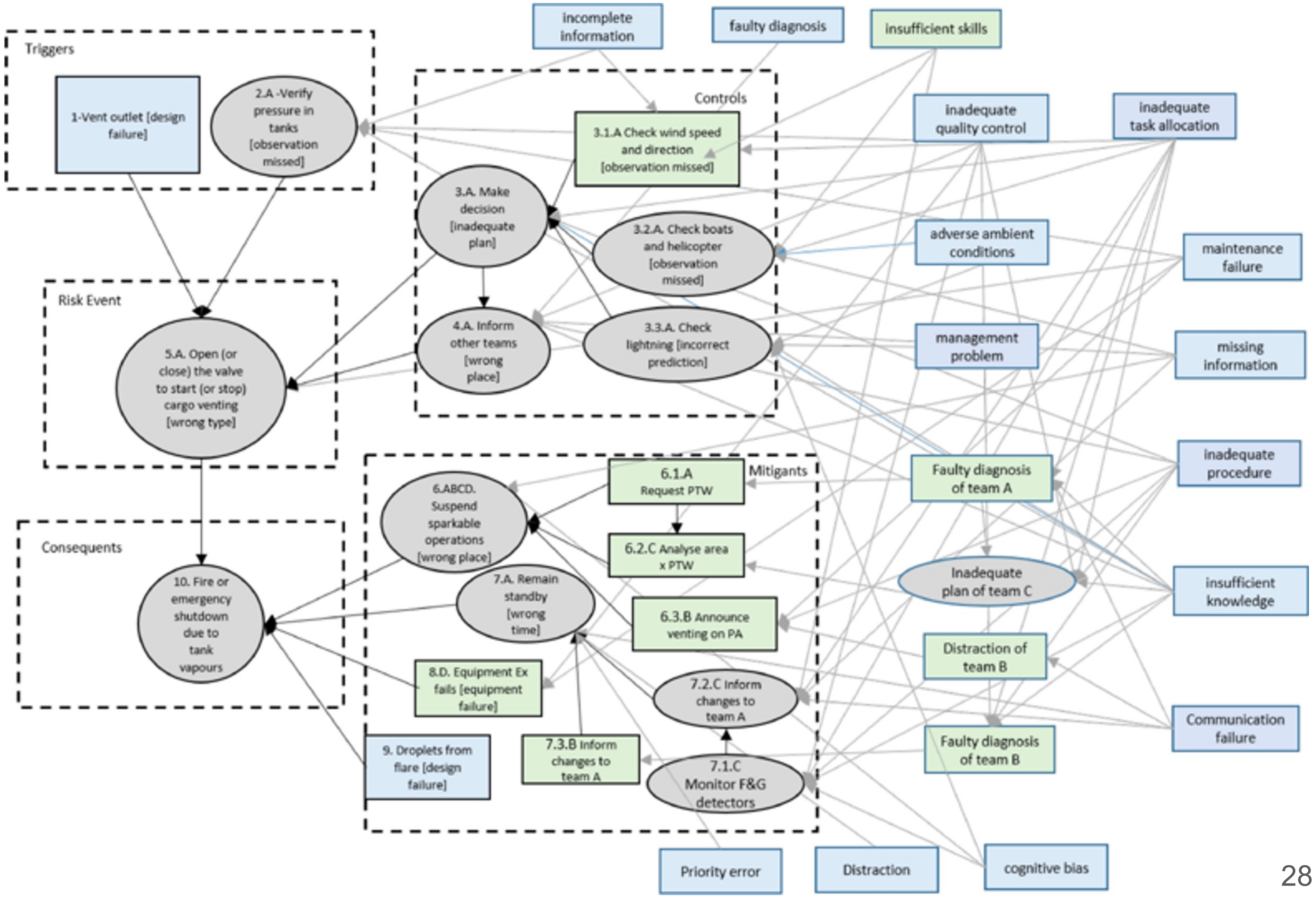
Identified Credal Network

Nodes identified with critical task analysis

Links defined with *cause-consequence idiom* which resembles the logic of a bow-tie diagram

discrete nodes: rectangles (child nodes in green, root nodes in blue)

credal nodes: grey ellipses



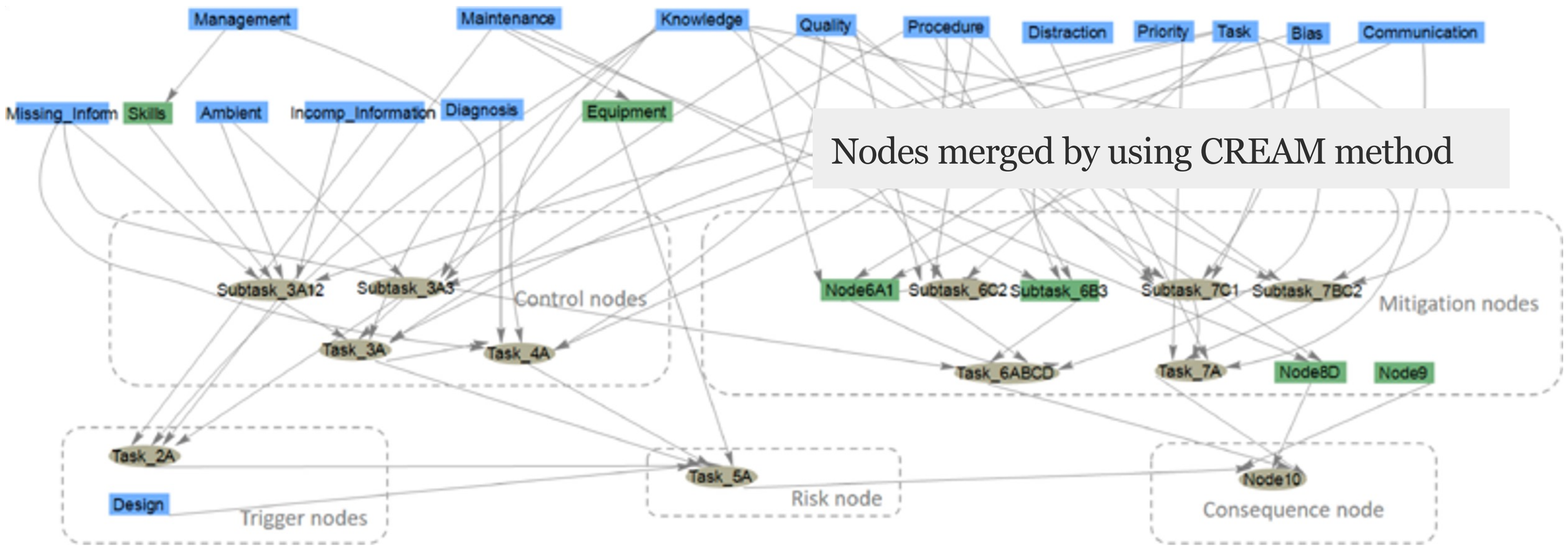
Identified Credal Network

Nodes identified with critical task analysis

Links defined with *cause-consequence idiom* which resembles the logic of a bow-tie diagram

Discrete nodes: rectangles (child nodes in green, root nodes in blue)

Credal nodes: grey ellipses

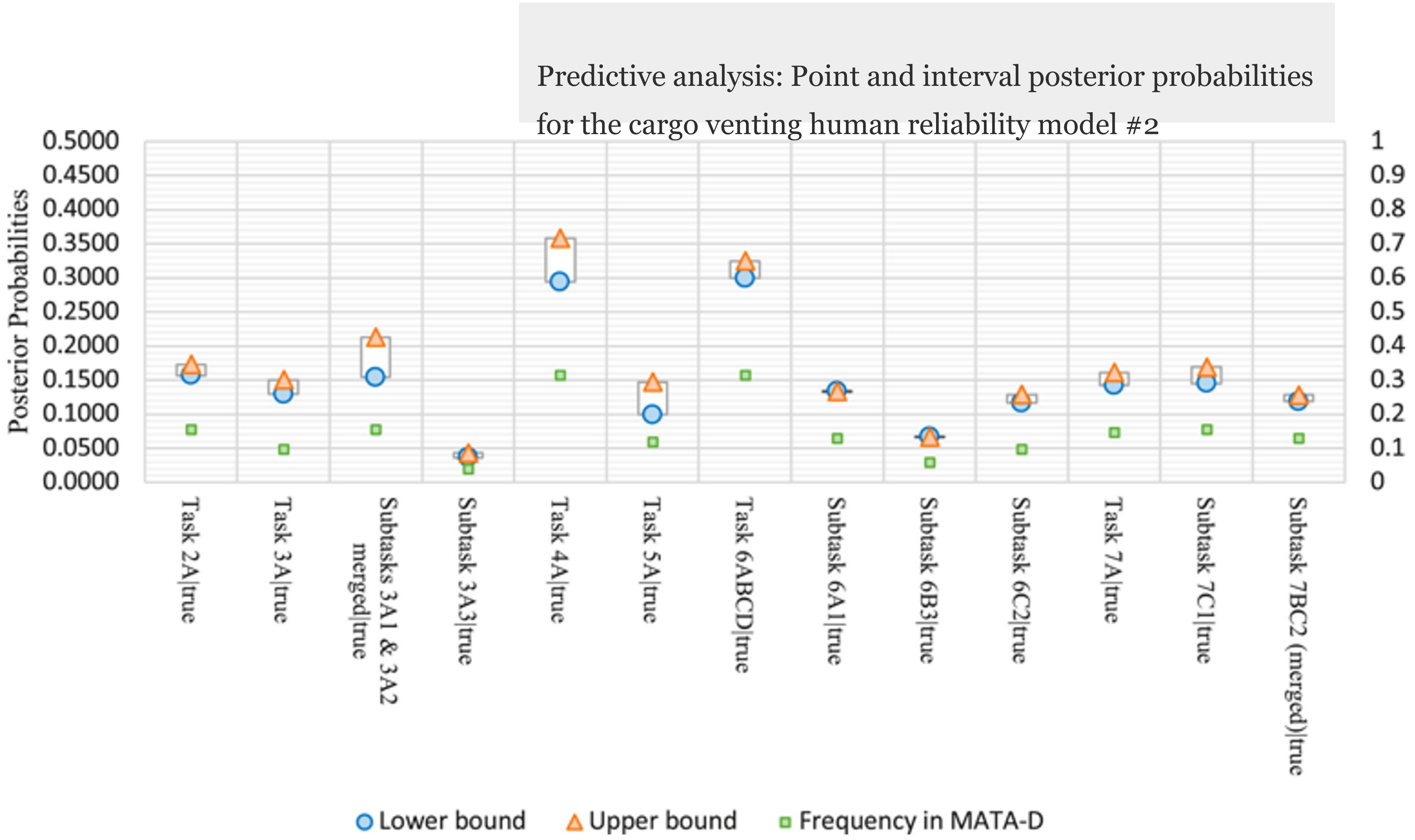


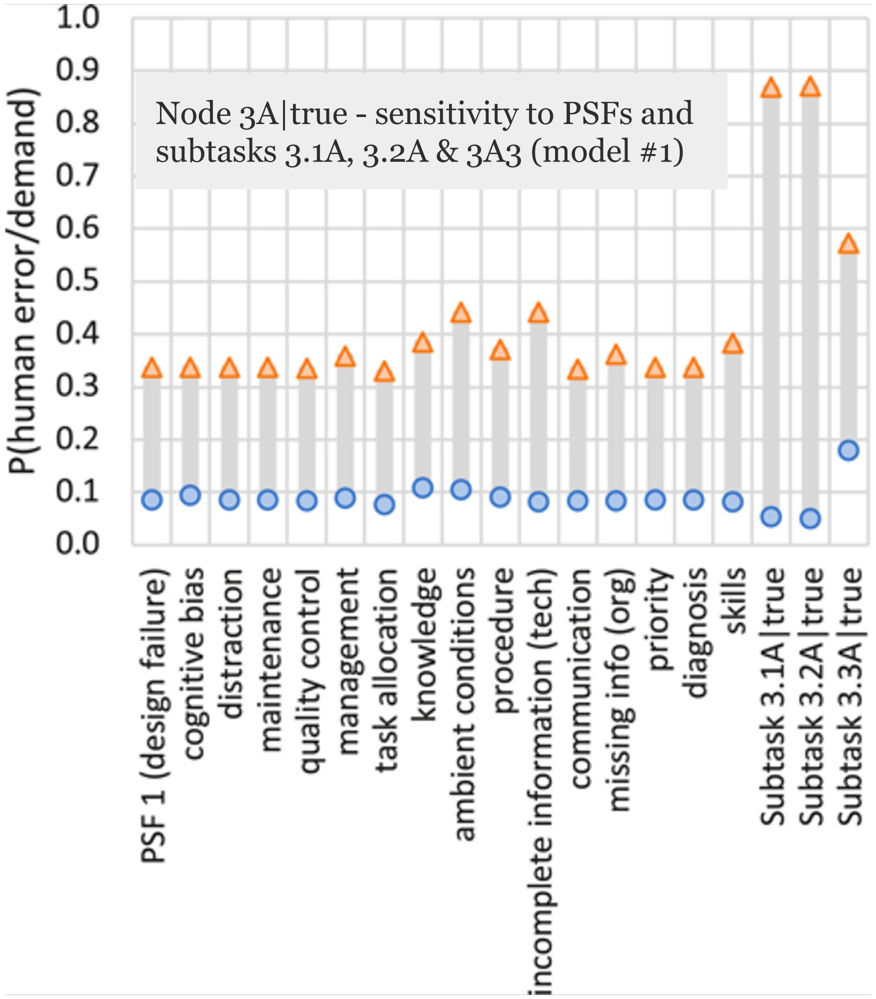
Prior probabilities expressed in terms of k out of n trials

Lack of possible combinations events in MATA-D **interpreted as missing data** rather than impossible events

Incomplete combinations replaced by **intervals [0,1]**

Selected results





Decision making and criteria selection

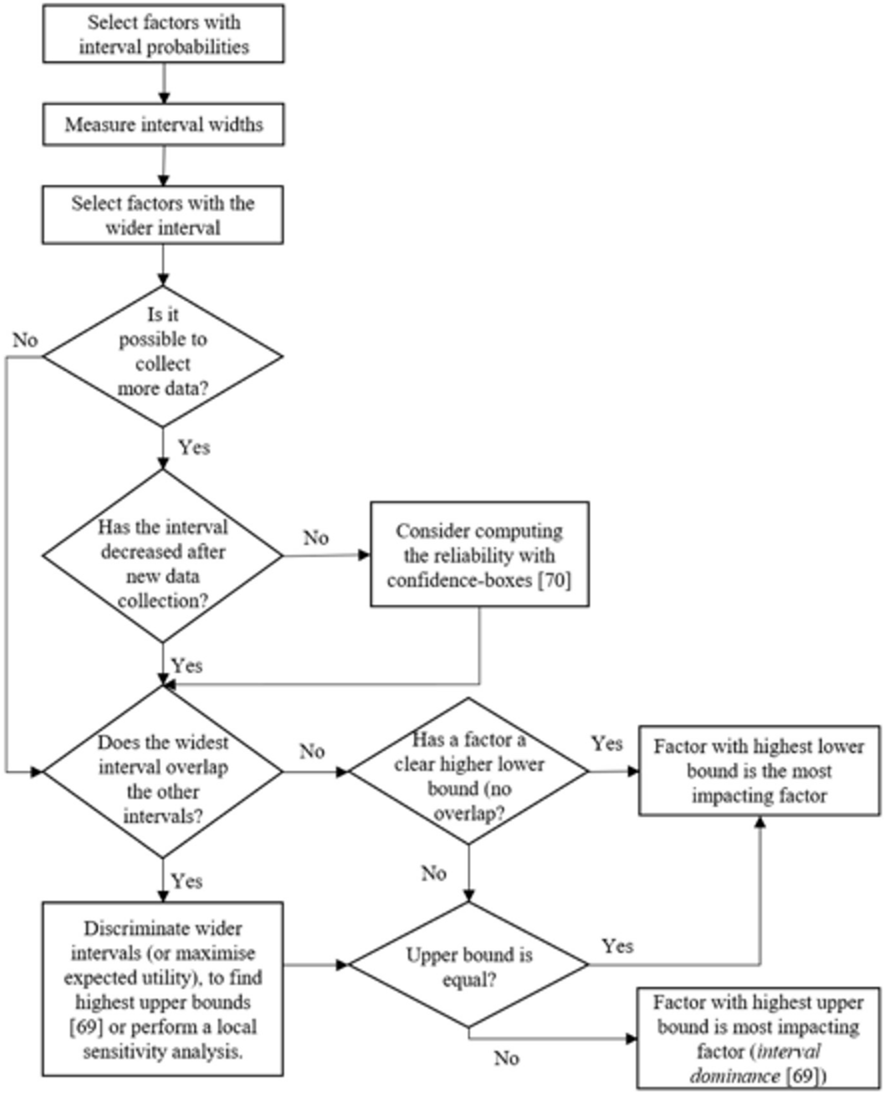
Output of the CN are intervals

Comparison of two or more variables affected by imprecision?

Selecting a factor $P(PSF1 = T)$ that reduce $P(HE = T)$

Different than reducing the imprecision of the conditional probability of the event, e.g., $P(HE{=}T|PSF1{=}T)$.

Define rules to select the most important factor under imprecision



Decision making and criteria selection

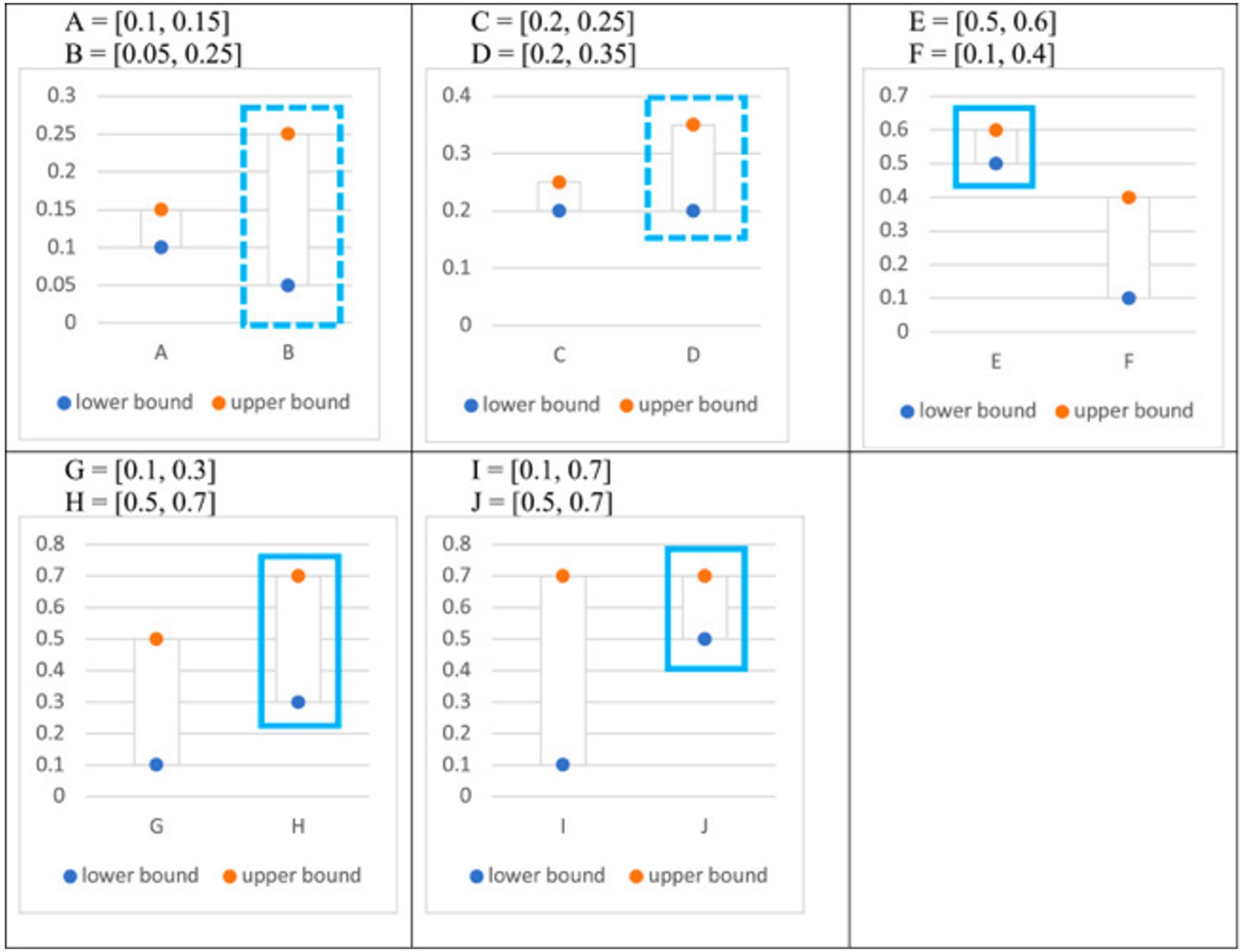
Output of the CN are intervals

Comparison of two of more variables affected by imprecision?

Selecting a factor *P(PSF1 = T)* that reduce *P(HE = T)*

Different than reducing the imprecision of the conditional probability of the event, e.g., *P(HE=T|PSF1=T)*.

Define rules to select the most important factor under imprecision



The data problem II (small data)

- Small sample sizes
- Data disproportion

(ignores important information regarding the evenness of sample sizes)

Communication failure	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
Inadequate task allocation	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	TRUE	TRUE
Insufficient knowledge	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE
Faulty diagnosis - FALSE	71	15	62	44	1	1	7	6
Faulty diagnosis - TRUE	1	1	7	12	2	3	3	2

Communication failure	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
Inadequate task allocation	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	TRUE	TRUE
Insufficient knowledge	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE
Faulty diagnosis – FALSE	0.99	0.94	0.90	0.79	0.33	0.25	0.70	0.75
Faulty diagnosis – TRUE	0.01	0.06	0.10	0.21	0.67	0.75	0.30	0.25

We need to understand if small sample sizes are “statistically significant”. Lack of transparency in Bayesian/Credal network to visualize the data disproportion issue.

Distortion model

Criticism to interval analysis ([0,1] interval for missing data):

- the lack of robustness (one observation may change from being the [0,1] interval to an exact relative frequency);
- a very conservative approach in some nodes and a precise approach in others,
- the use of the vacuous model [0,1] is equivalent to applying *natural extension* in the imprecise probability literature

P_o estimated from the available data

δ related to the proportion of noisy data (probability that any other probability measure is possible)

Safety Science
Volume 157, January 2023, 105915

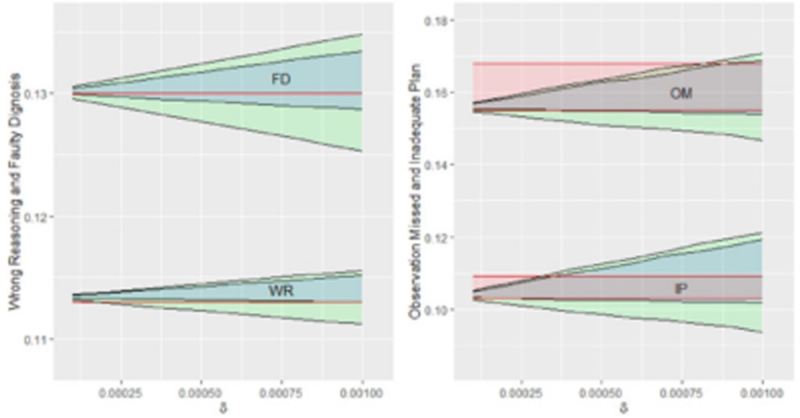
Distortion models for estimating human error probabilities

Pablo-Ramsés Alonso-Martín [a], Ignacio Montes [b], Enrique Miranda [b]

$$\underline{P}_{LV}(A) = \begin{cases} (1-\delta)\, P_0(A) & \text{if } A \neq \mathscr{X}. \\ 1 & \text{if } A = \mathscr{X}. \end{cases}$$

credal set

$$\mathscr{M}(\underline{P}_{LV}) = \{(1-\delta)\, P_0 + \delta P \mid P \text{ probability measure}$$

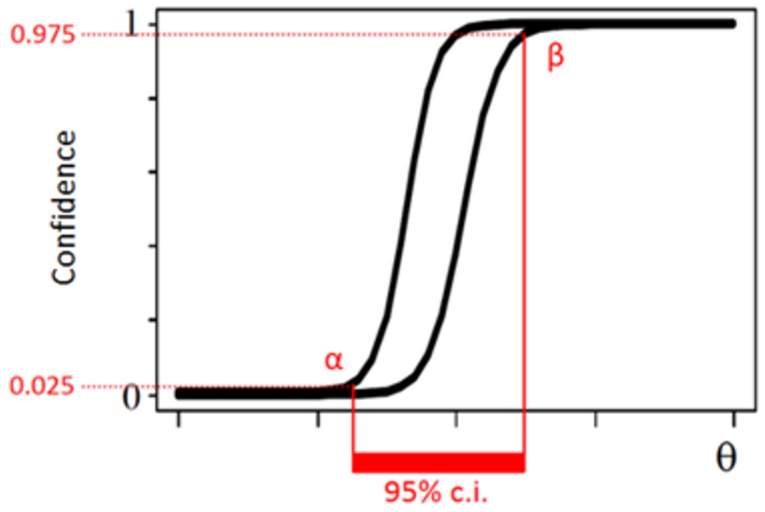


Estimated HEP (Faulty Diagnosis (FD), Wrong Reasoning (WR), Observation Missed (OM) and Inadequate Plan (IP)) using the approach from (Morais et al., 2019a, Morais et al., 2019b) (in red), the linear vacuous model (in blue) and the total variation model (in green).

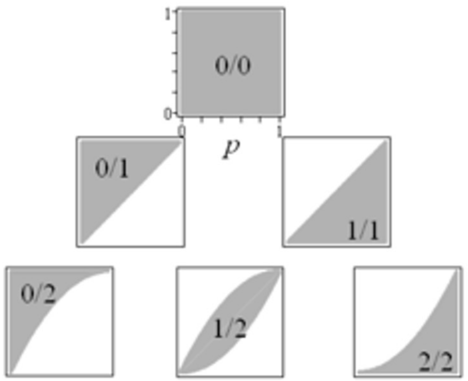
Confidence boxes (c-boxes)

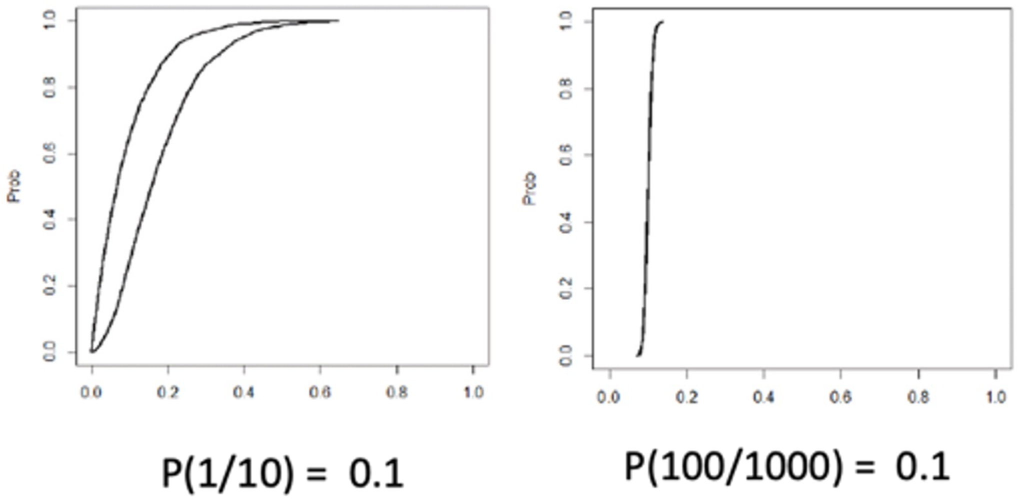
Generalisations of statistical confidence distributions
Encode confidence intervals at every confidence level

The bases for c-boxes are the classical notions of confidence (Neyman, 1937), confidence distributions (Cox, 1958), imprecise probability concepts (Walley, 1991) and probability boxes (Ferson et al., 2003).



Redrawn from https://sites.google.com/site/confidenceboxes/

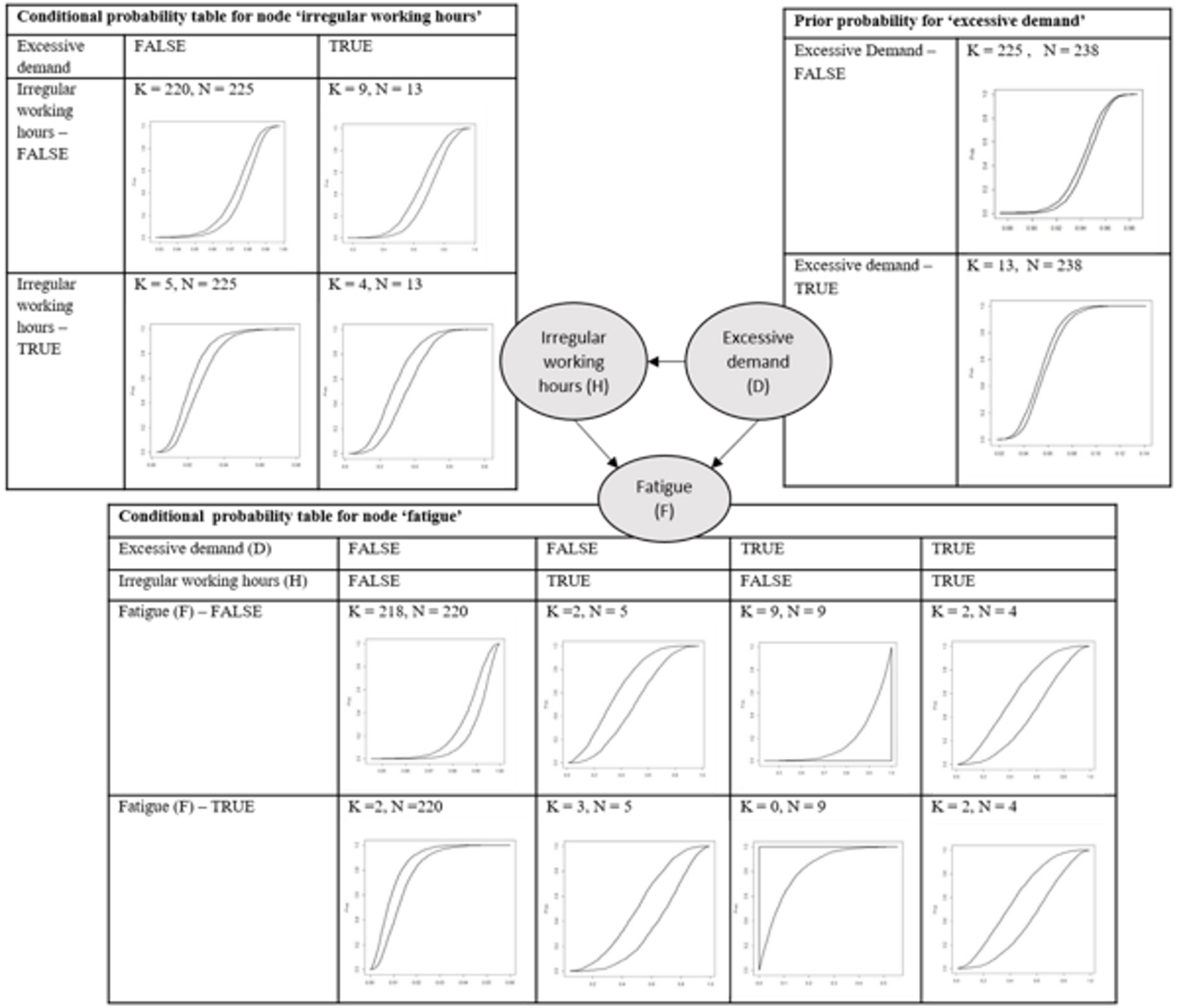




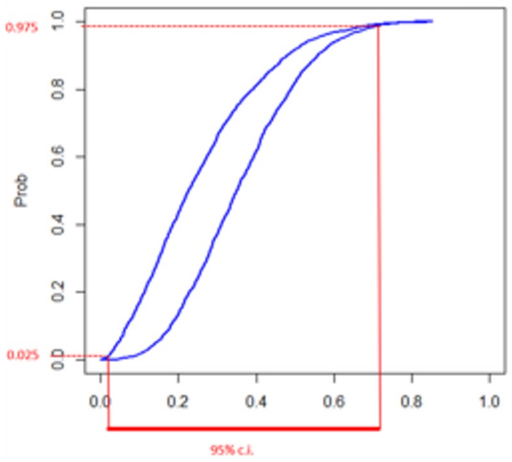
Credal network with c-boxes

No additional parameters required

- imprecision arising from small data sets can be propagated through the model with c-boxes
- Return the results in terms of (desired) confidence interval



P(D|F)



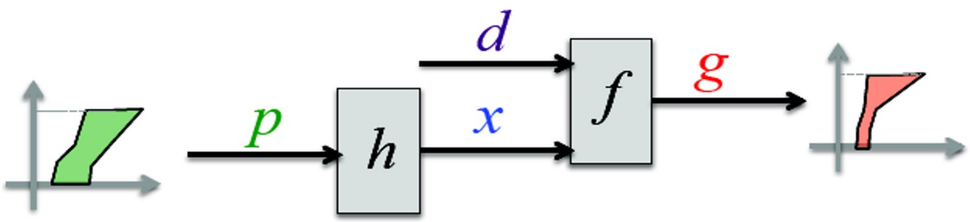
C.Morais, S.Ferson, R.Moura, S.Tolo, M.Beer, E.Patelli. **2021** 'Handling the Uncertainty with Confidence in Human Reliability Analysis'. In Proceedings of the 31st European Safety and Reliability Conference. Angers, France: Research Publishing, Singapore. https://doi.org/10.3850/978-981-18-2016-8 357-cd.

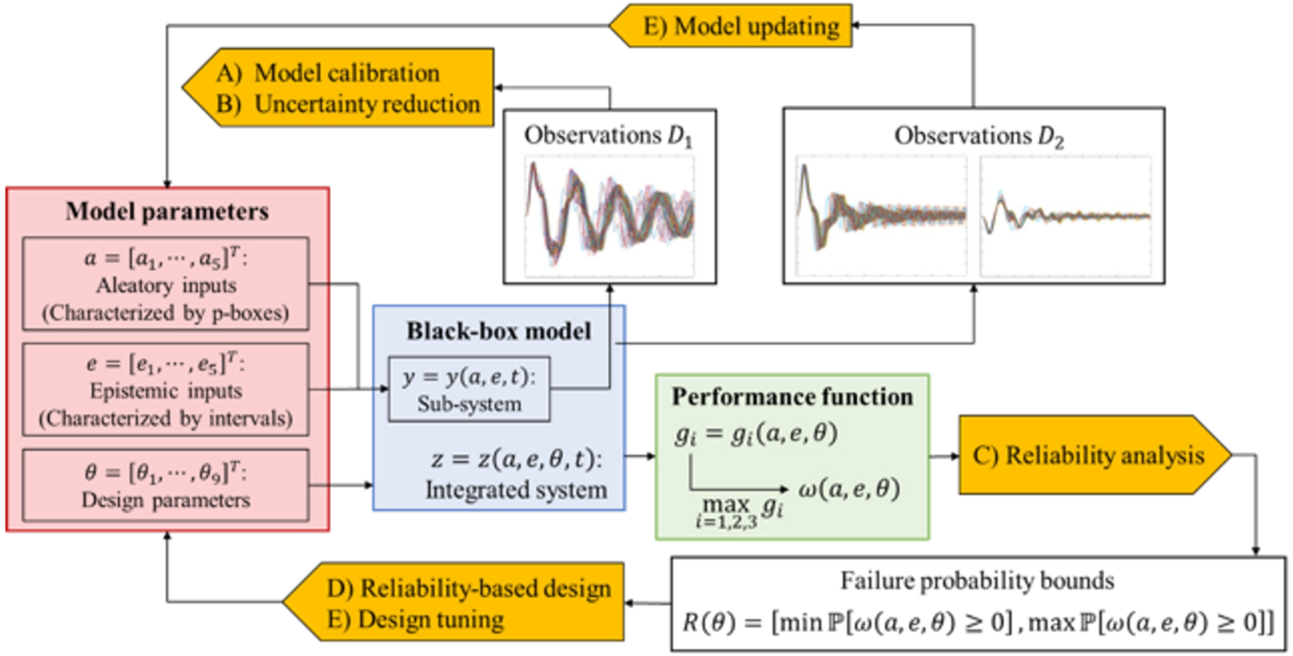
NASA UQ Challenge problem

2019

Computational challenges

Propagate and characterize uncertainty and incertitude of a black-box





- Patelli, E., Alvarez, D.A., Broggi, M., de Angelis, M., 2015. Uncertainty management in multidisciplinary design of critical safety systems. J. of Aerospace Information Systems 12, 140–169 https://doi.org/10.2514/1.I010273
- Lye, A., Broggi, M., Kitahara, M., Patelli, E. 2022. Robust optimization of a dynamic Black-box system under severe uncertainty: A distribution-free framework, Mechanical Systems and Signal Processing 167A, https://doi.org/10.1016/j.ymssp.2021.108522

Model updating subproblem (Task A)

5 aleatory inputs $a \sim f_a$

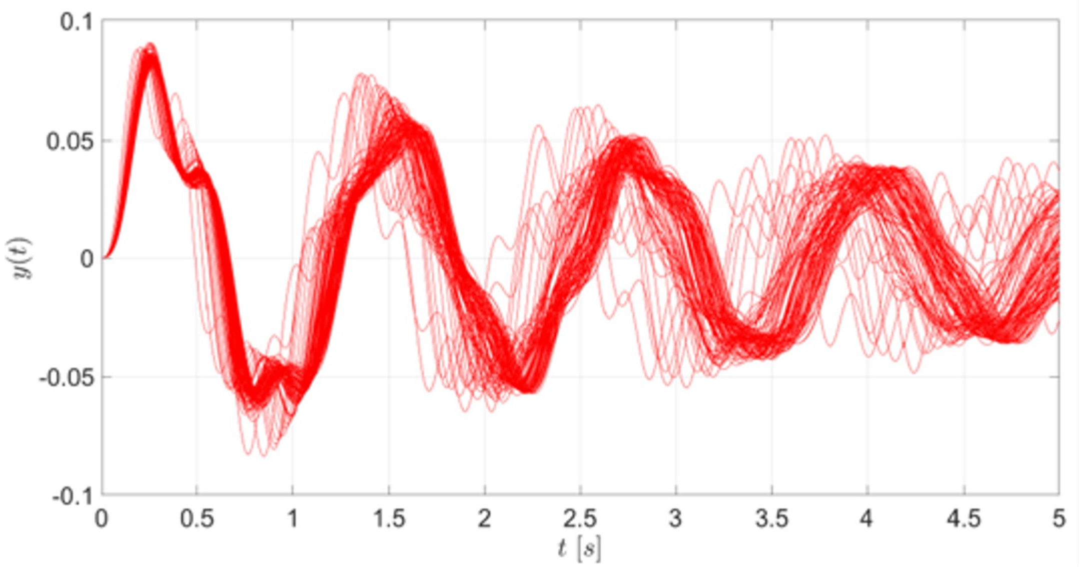
- Distribution family is unknown a priori
- Support domain is given: $a \in [0, 2]^5$

4 epistemic inputs $e \sim E$

- Support domain is given: $e \in [0, 2]^4$

100 sets of observations:

- $y = y(\mathbf{a}, \mathbf{e}, t)$,
- $z = z(a, e, \theta, t)$



Challenges (adapted from Matteo Broggi)

Model updating under hybrid uncertainties

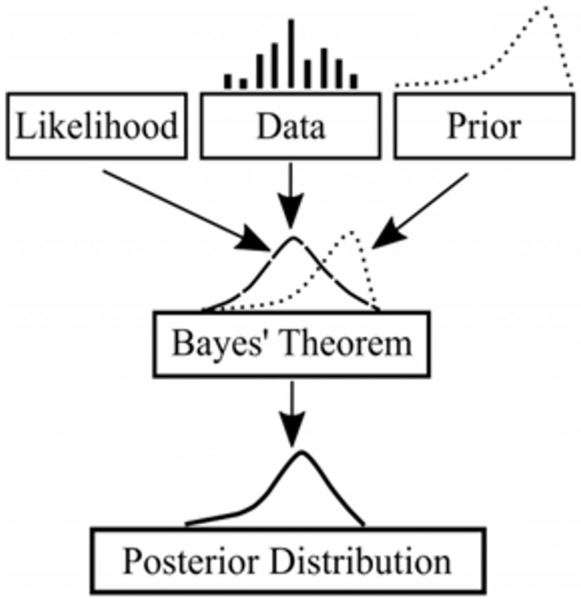
§

- Hypotheses on the distribution families of aleatory inputs;
- Dependence structure among aleatory inputs;
- Updating dynamic systems: very high dimensional observations.

Enhancement of subjective assumption-free framework

- How to calibrate the PDF of aleatory inputs whose distribution families are unknown?
- How to calibrate the correlated joint PDF?
- How to incorporate the time dependent observations to update the dynamic systems?

Our solution for Task A



Bayesian model updating approach

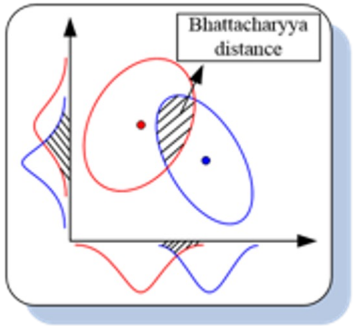
- Aleatory variables modelled as a staircase density functions
- Bhattacharyya distance-based

$$d = -\log\left[\int_{-\infty}^{\infty}\sqrt{P_{\boldsymbol{D}}(x)\cdot P_{\hat{y}}(x)}\,dx\right]$$

- Approximate Bayesian Computation,

$$P(\boldsymbol{D}|\boldsymbol{\Theta}, M) \propto \exp\left(-\frac{d_B^2}{\epsilon_B^2}\right)$$

- Comparing the time series through a moving window procedure
- Totally 4+5×4=24 epistemic parameters are updated



Distributional overlap distance

Staircase Random Variable

(adapted from Matteo Broggi)

Proposed by Crespo et al 2018 https://doi.org/10.1016/j.apm.2018.07.029

Univariate random variable having:

- Bounded support set: Ω_x and first four moments: $\boldsymbol{\theta}$

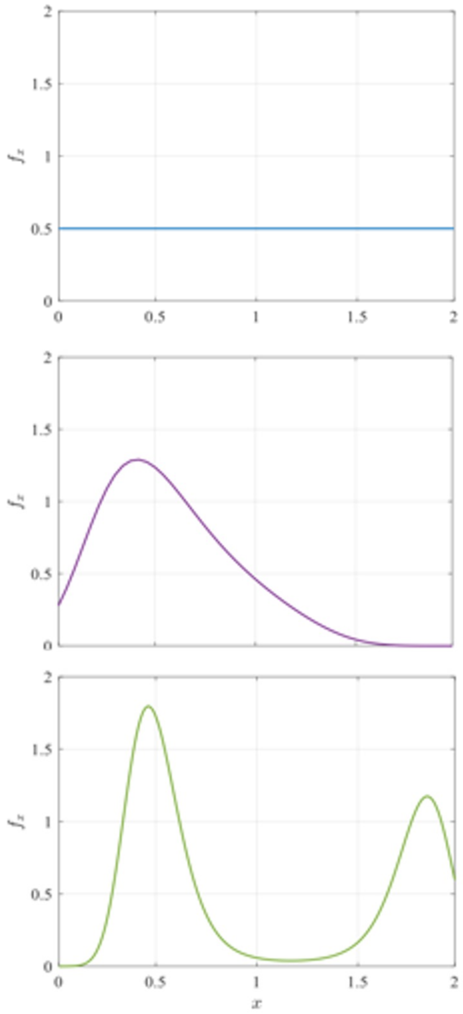
Staircase density function

- Piecewise constant function: $f_x(x) = l_i \; \forall x \in (x_i, x_{i+1}], \text{for } 1 \leq i \leq n_b$
- n_b – number of bins

Examples

- Ω_x= [0, 2], $\boldsymbol{\theta}$=[1.0, 0.33, 0, 1.8] Uniform distribution
- Ω_x= [0, 2], $\boldsymbol{\theta}$=[0.57, 0.10, 0.59, 2.86] Beta distribution (left skewed)
- Ω_x= [0, 2], $\boldsymbol{\theta}$=[1.0, 0.42 0.42, 1.37] Bi-modal distribution

Characterize aleatory inputs whose distribution families are unknown



Staircase Random Variable

(adapted from Matteo Broggi)

Optimization problem on moment matching constraints:

$$\hat{\boldsymbol{l}} = \underset{\boldsymbol{l}\geq 0}{\operatorname{argmin}}\{J(\boldsymbol{l}):\ \mathbf{A}(\boldsymbol{\theta}, n_b)\boldsymbol{l} = \mathbf{b}(\boldsymbol{\theta}), \boldsymbol{\theta} \in \Theta\}$$

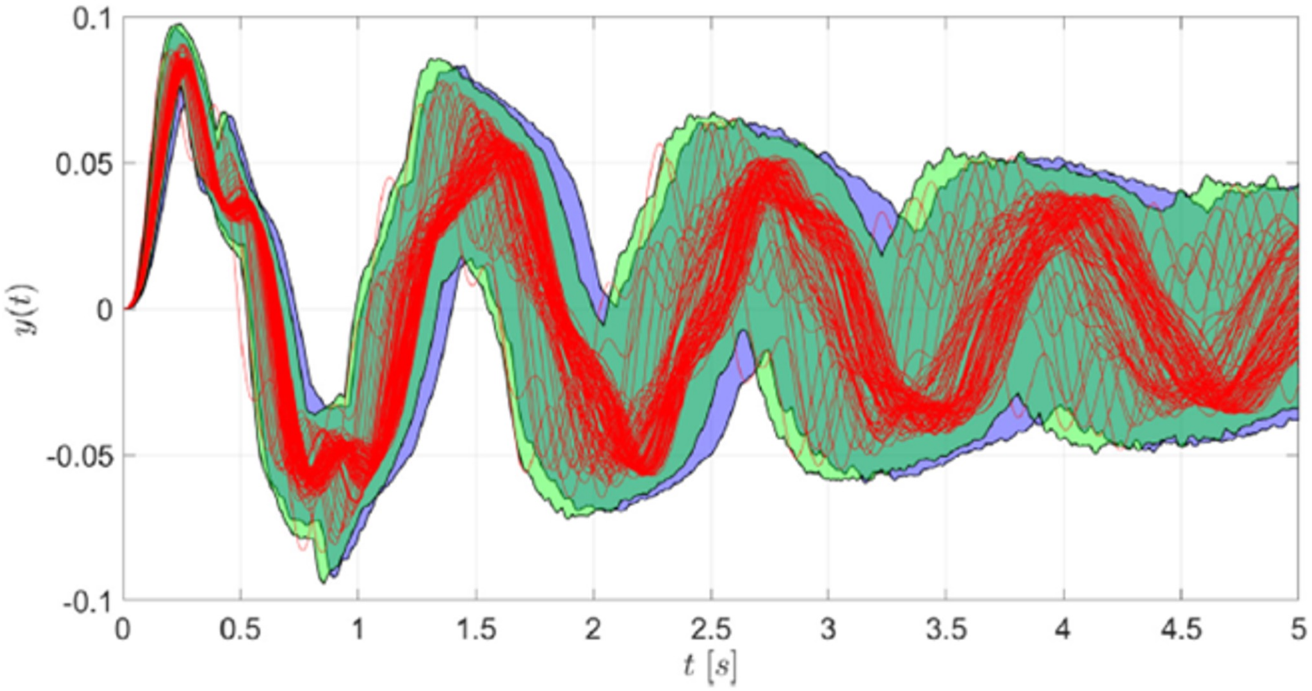
*J(**l**)* – cost function, e.g., maximum entropy;

Θ – **θ** feasible domain, $g_i(\boldsymbol{\theta})\leq 0$, i=1,...,13

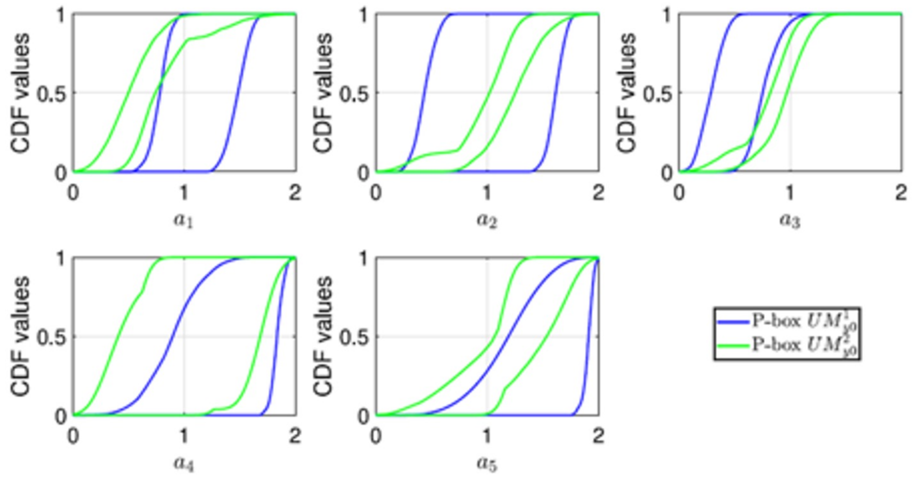
	Moment constraints		Moment constraints
Mean μ_i	$g_1 = \underline{x}_i - \mu_i$	Kurtosis $\tilde{m}_{4i}$	$g_{10} = -\tilde{m}_{4i} m_{2i}^2$
	$g_2 = \mu_i - \overline{x}_i$		$g_{11} = 12\tilde{m}_{4i} m_{2i}^2 - (\overline{x}_i - \underline{x}_i)^4$
Variance m_{2i}	$g_3 = -m_{2i}$		$g_{12} = \left(\tilde{m}_{4i} m_{2i}^2 - v_i m_{2i} - u_i \tilde{m}_{3i} m_{2i}^{3/2}\right)(v_i - m_{2i}) + \left(\tilde{m}_{3i} m_{2i}^{3/2} - \mu_i m_{2i}\right)^2$
	$g_4 = m_{2i} - v_i$		
Skewness $\tilde{m}_{3i}$	$g_5 = m_{2i}^2 - m_{2i}(\mu_i - \underline{x}_i)^2 - \tilde{m}_{3i} m_{2i}^{3/2}(\mu_i - \underline{x}_i)$		$g_{13} = \tilde{m}_{3i}^2 m_{2i}^3 + m_{2i}^3 - \tilde{m}_{4i} m_{2i}^3$
	$g_6 = \tilde{m}_{3i} m_{2i}^{3/2}(\overline{x}_i - \mu_i) - m_{2i}(\overline{x}_i - \mu_i)^2 + m_{2i}^2$		
	$g_7 = 4m_{2i}^2 + \tilde{m}_{3i}^2 m_{2i}^3 - m_{2i}^2(\overline{x}_i - \underline{x}_i)^2$		
	$g_8 = 6\sqrt{3}\tilde{m}_{3i} m_{2i}^{3/2} - (\overline{x}_i - \underline{x}_i)^3$		
	$g_9 = -6\sqrt{3}\tilde{m}_{3i} m_{2i}^{3/2} - (\overline{x}_i - \underline{x}_i)^3$		

Selected results (Task A)

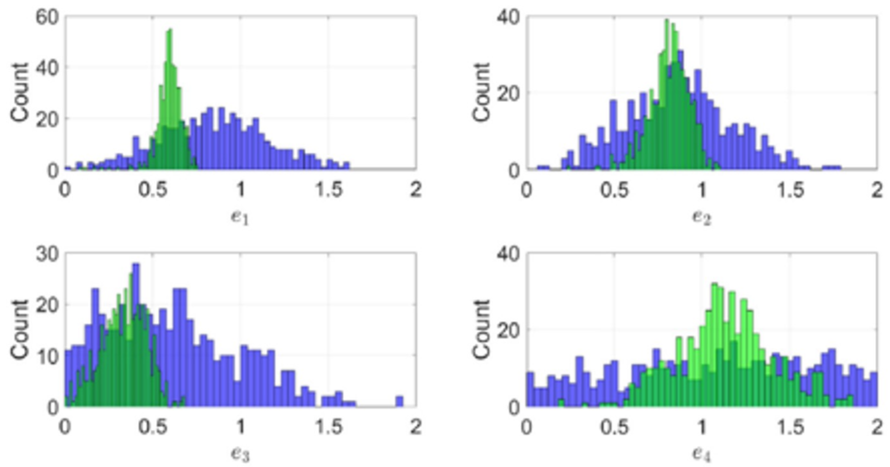
Blue: Distribution-based approach (Beta distributions)
Green: Distribution-free approach
Red: Experimental data



Realization of the uncertainty model *y*



Posterior distributions of ***a***



samples of ***e***

University of Strathclyde Glasgow

Lessons learned*

* The comments are not referring to a specific solution or methodology

ELSEVIER
Mechanical Systems and Signal Processing
Volume 164, 1 February 2022, 108253

Synthetic Validation of Responses to the NASA Langley Challenge on Optimization under Uncertainty

Luis G. Crespo, Sean P. Kenny

From Crespo and Kenny

"**The quantification of aleatory uncertainty was poor overall**, whereas the quantification of the epistemic uncertainty was generally acceptable."

"(system) knowledge basis is essential to deal with epistemic uncertainty" and "[...] uncertainty reduction in series rather than in parallel."

"**Parameter dependencies** in the aleatory variables were **grossly mischaracterized.**"

"Robust design optimization overcame the shortcomings of poor uncertainty modeling."

"If the epistemic scenarios being ignored turn out to be infeasible, the performance improvements resulting from the risk-based design will be well justified. Otherwise, the possibly gross underestimation of the failure probability might render the UQ modeling and design optimization processes not only pointless but also liable"

"This outcome [of the challenge problem] highlights the need for developing and maturing the processes needed to effectively model and manage the effects of uncertainty in model predictions."

Computational tools (Ander Gray)

Probability Theory

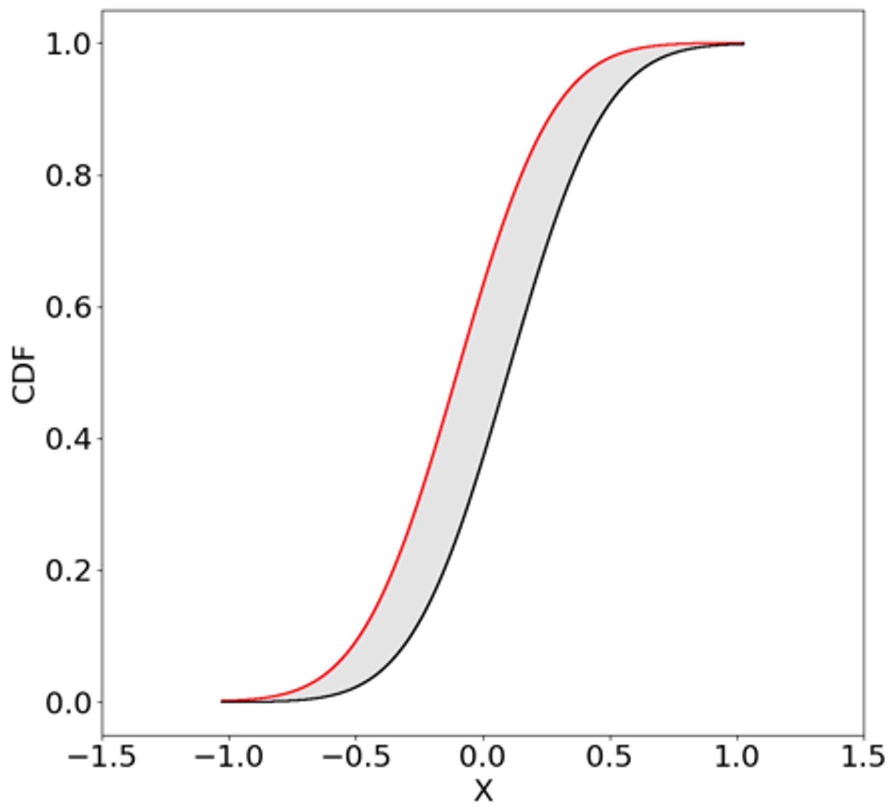
Interval Arithmetic

ProbabilityBoundsAnalysis.jl (p-box arithmetic)

MomentArithmetic.jl (moment arithmetic)

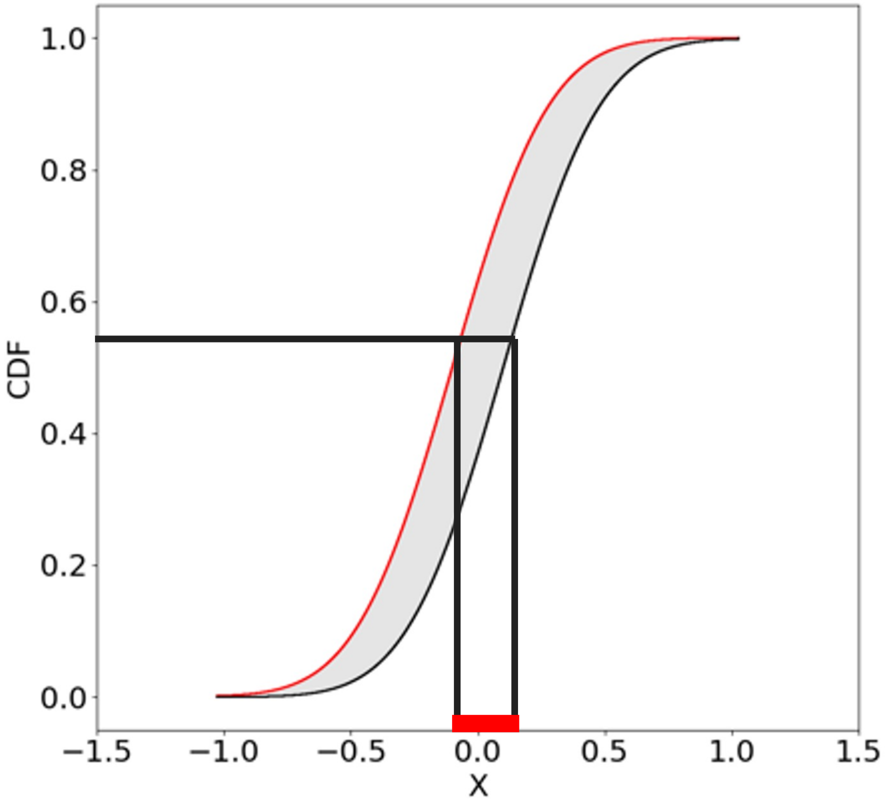
ZoneArithmetic.jl (Subintevalization)

Probability boxes



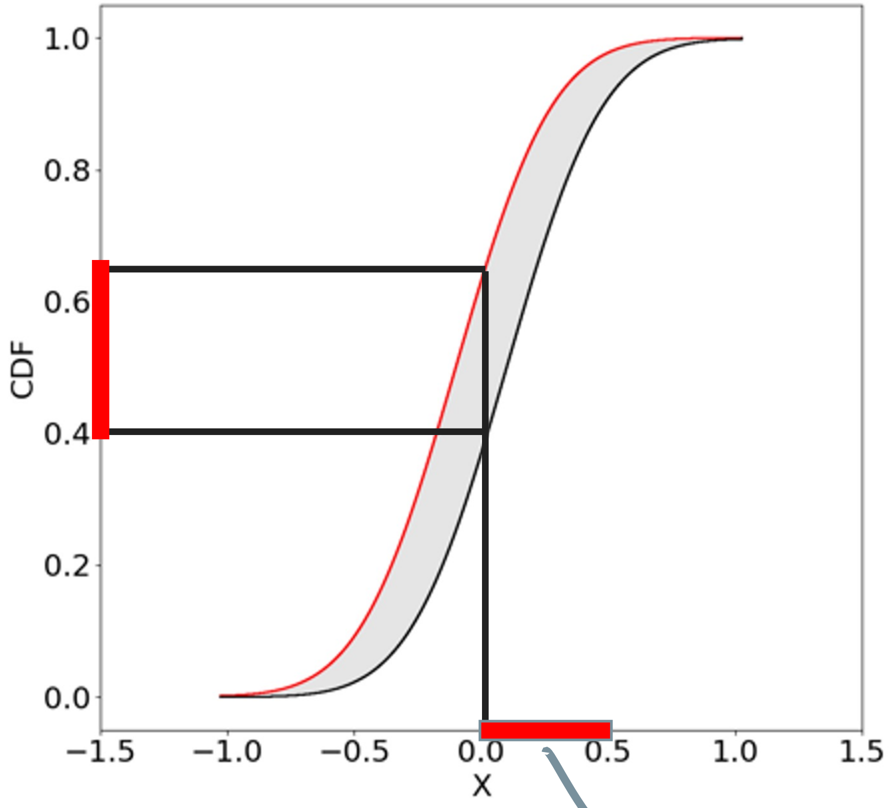

```
               _
   _       _ _(_)_     |  Documentation: https://docs.julialang.org
  (_)     | (_) (_)    |
   _ _   _| |_  __ _   |  Type "?" for help, "]?" for Pkg help.
  | | | | | | |/ _` |  |
  | | |_| | | | (_| |  |  Version 1.6.0 (2021-03-24)
 _/ |\__'_|_|_|\__'_|  |  Official https://julialang.org/ release
|__/                   |

julia> using ProbabilityBoundsAnalysis

julia> pb1 = normal(interval(-0.1,0.1), 0.3)
Pbox:      ~ normal ( range=[-1.0271, 1.0271], mean=[-0.1, 0.1], var=0.09)

julia> plot(pb1)
```

Probability boxes




```
               _
   _       _ _(_)_     |  Documentation: https://docs.julialang.org
  (_)     | (_) (_)    |
   _ _   _| |_  __ _   |  Type "?" for help, "]?" for Pkg help.
  | | | | | | |/ _` |  |
  | | |_| | | | (_| |  |  Version 1.6.0 (2021-03-24)
 _/ |\__'_|_|_|\__'_|  |  Official https://julialang.org/ release
|__/                   |

julia> using ProbabilityBoundsAnalysis

julia> pb1 = normal(interval(-0.1,0.1), 0.3)
Pbox:     ~ normal ( range=[-1.0271, 1.0271], mean=[-0.1, 0.1], var=0.09)

julia> plot(pb1)

julia> cut(pb1, 0.55)
[-0.0623016, 0.141492]

julia> rand(pb1, 10^3)
1000-element Vector{Interval{Float64}}:
  [0.21093, 0.417437]
 [-0.363369, -0.157885]
 [-0.09624, 0.107521]
  [0.379457, 0.593457]
  [0.204566, 0.410931]
 [-0.0470878, 0.156736]
  [0.204566, 0.410931]
 [-0.288402, -0.0838438]
 [-0.392235, -0.186249]
  [0.0573201, 0.261651]
```

Probability boxes



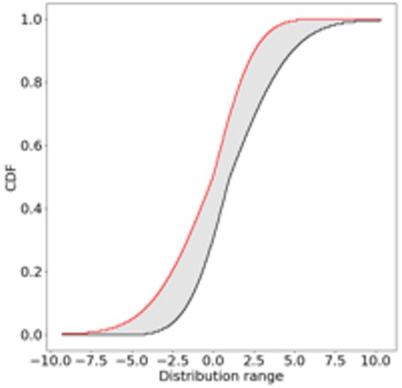

```
julia> cdf(pb1, 0)
[0.364999, 0.635001]

julia> mass(pb1, interval(0, 0.5) )
[0.344999, 0.615]
```

$$\mathbb{P}_X = [0.345, 0.615]$$

Distributions with imprecise parameters

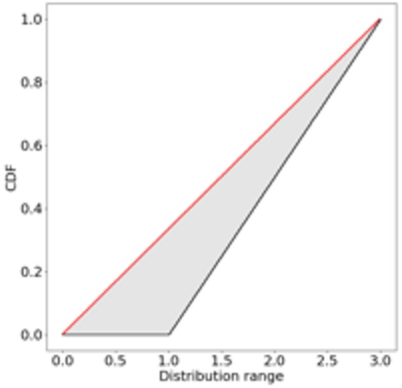
Normal

$\mu \in [0,1], \sigma \in [2,3]$



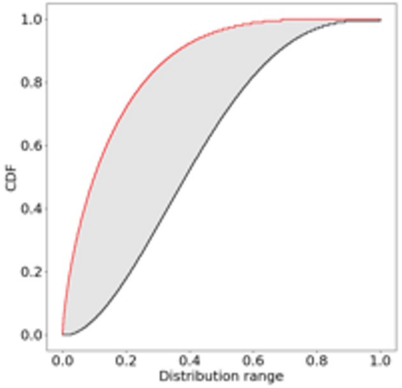
Uniform

$U_1 \in [0,1], U_2 = 3$



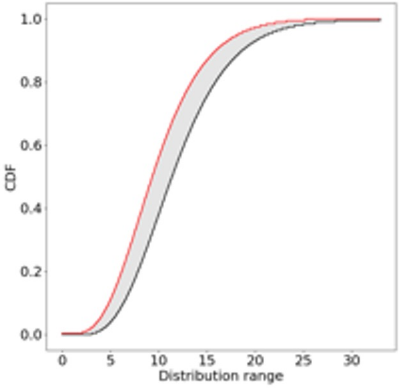
Beta

$\alpha \in [0.7, 1], \beta \in [3,4]$



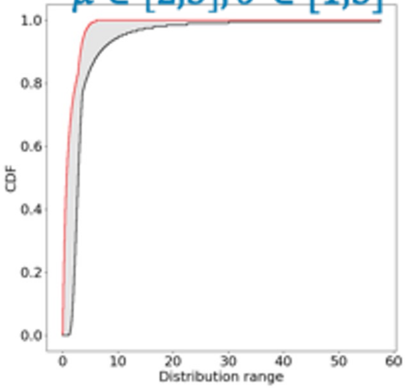
Gamma

$\alpha \in [5, 6], \beta = 2$



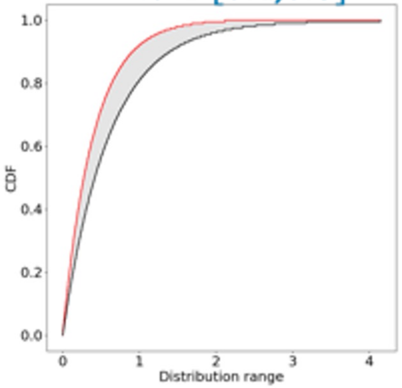
Lognormal

$\mu \in [2,3], \sigma \in [1,5]$



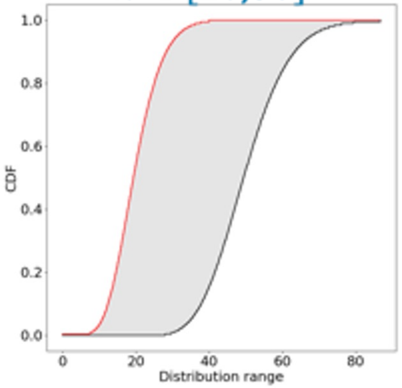
Exponential

$\lambda \in [0.4,0.6]$



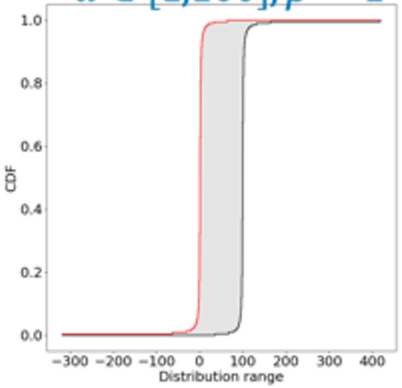
Chi squared

$k \in [20,50]$

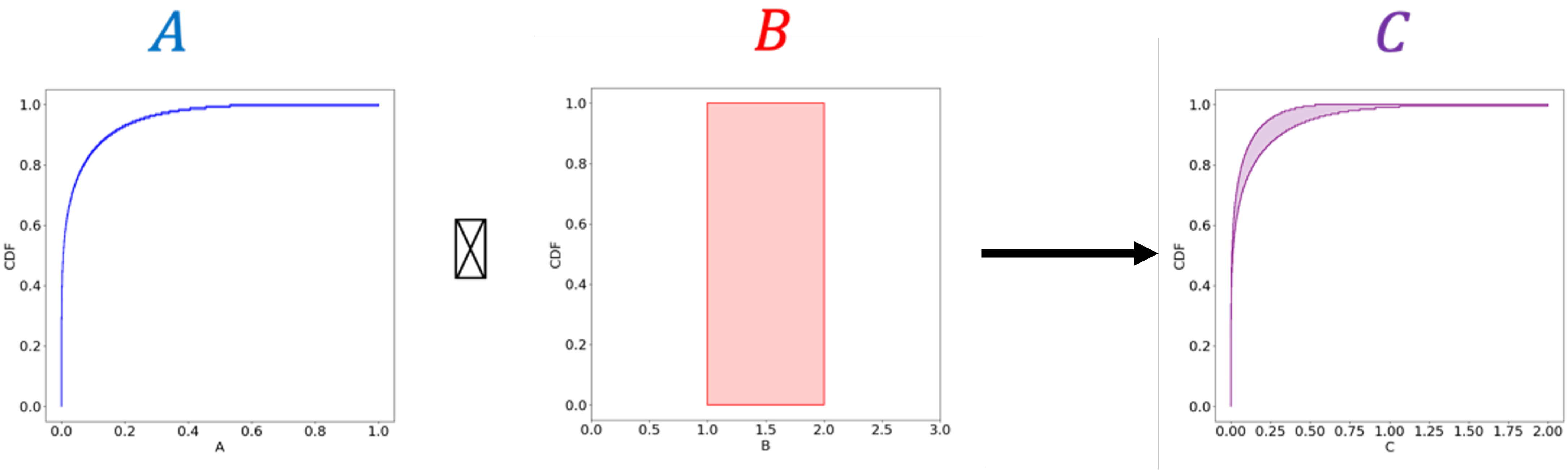


Cauchy

$\alpha \in [1,100], \beta = 1$



Binary operations



Risk analysis

What if we do not have distributions?

Ander Gray Scott Ferson Vladik Kreinovich Edoardo Patelli

First order error propagation

- In probability theory called "*Moment propagation*"
- Propagates mean & variance through mathematical expressions
- Although widely used, requires a lot of assumptions, and does not inform about distribution shape

- Requires stochastic independence
- Moments must be perfectly known
- Gives no information about tails without distribution assumption
- Only accurate for near-linear models

Bad for risk analysis

Pessimism from Cullen and Frey (1999)

Moment arithmetic

Generalisation with the following:

- Independence between variables need not be assumed
- Moment propagation formulae may be evaluated with intervals
- Assumptions about input distributions is no longer necessary
- Works for non-linear functions

Computer arithmetic on random variable's ***range, mean*** and ***variance***

7 binary operations: +, -, *, /, ^, min, max

9 unary operations: rescale, shift, exp, log, ln, 1/x, x^2, $\sqrt{x}$, $|x|$

For independence and unknown dependence (Fréchet)

Gray, Ander, Scott Ferson, Vladik Kreinovich, and Edoardo Patelli. 'Distribution-Free Risk Analysis'. International Journal of Approximate Reasoning 146 (2022): 133–56. https://doi.org/10.1016/j.ijar.2022.04.001.

Moment arithmetic

Rigorous formulas for least and greatest possible values

Interval arithmetic

	Least possible value	*Greatest possible value*
$k+X$ (shifting)	$\mathbf{k}+\underline{\mathbf{X}}$	$\mathbf{k}+\overline{\mathbf{X}}$
kX (rescaling)	$\begin{cases}\mathbf{k}\underline{\mathbf{X}}, & \text{if } 0 \le k \\ \mathbf{k}\overline{\mathbf{X}}, & \text{if } k < 0\end{cases}$	$\begin{cases}\mathbf{k}\overline{\mathbf{X}}, & \text{if } 0 \le k \\ \mathbf{k}\underline{\mathbf{X}}, & \text{if } k < 0\end{cases}$
e^X	$\mathbf{e}^{\underline{\mathbf{X}}}$	$\mathbf{e}^{\overline{\mathbf{X}}}$
$\ln(X)$ for $0 < X$	$\mathbf{ln}(\underline{\mathbf{X}})$	$\mathbf{ln}(\overline{\mathbf{X}})$
$\log_{10}(X)$ for $0 < X$	$\mathbf{log_{10}}(\underline{\mathbf{X}})$	$\mathbf{log_{10}}(\overline{\mathbf{X}})$
$\frac{1}{X}$ for $0 \notin X$	$\mathbf{1}/\overline{\mathbf{X}}$	$\mathbf{1}/\underline{\mathbf{X}}$
X^2	$\begin{cases}\mathbf{0}, & \text{if } 0 \in X \\ \mathbf{min}(\underline{\mathbf{X}}^2, \overline{\mathbf{X}}^2), & \text{otherwise}\end{cases}$	$\mathbf{max}(\underline{\mathbf{X}}^2, \overline{\mathbf{X}}^2)$
$\lvert X\rvert$ (absolute value)	$\begin{cases}\mathbf{0}, & \text{if } 0 \in X \\ \mathbf{min}(\lvert\underline{\mathbf{X}}\rvert, \lvert\overline{\mathbf{X}}\rvert), & \text{otherwise}\end{cases}$	$\mathbf{max}(\lvert\underline{\mathbf{X}}\rvert, \lvert\overline{\mathbf{X}}\rvert)$
$\sqrt{X}$ for $0 \le X$	$\sqrt{\underline{\mathbf{X}}}$	$\sqrt{\overline{\mathbf{X}}}$
$X+Y$	$\underline{\mathbf{X}}+\underline{\mathbf{Y}}$	$\overline{\mathbf{X}}+\overline{\mathbf{Y}}$
$X-Y$	$\underline{\mathbf{X}}-\overline{\mathbf{Y}}$	$\overline{\mathbf{X}}-\underline{\mathbf{Y}}$
$X \times Y$	$\mathbf{min}(\underline{\mathbf{X}}\underline{\mathbf{Y}}, \underline{\mathbf{X}}\overline{\mathbf{Y}}, \overline{\mathbf{X}}\underline{\mathbf{Y}}, \overline{\mathbf{X}}\overline{\mathbf{Y}})$	$\mathbf{max}(\underline{\mathbf{X}}\underline{\mathbf{Y}}, \underline{\mathbf{X}}\overline{\mathbf{Y}}, \overline{\mathbf{X}}\underline{\mathbf{Y}}, \overline{\mathbf{X}}\overline{\mathbf{Y}})$
$\frac{X}{Y}$ for $0 \notin Y$	$\mathbf{min}(\underline{\mathbf{X}}/\underline{\mathbf{Y}}, \underline{\mathbf{X}}/\overline{\mathbf{Y}}, \overline{\mathbf{X}}/\underline{\mathbf{Y}}, \overline{\mathbf{X}}/\overline{\mathbf{Y}})$	$\mathbf{max}(\underline{\mathbf{X}}/\underline{\mathbf{Y}}, \underline{\mathbf{X}}/\overline{\mathbf{Y}}, \overline{\mathbf{X}}/\underline{\mathbf{Y}}, \overline{\mathbf{X}}/\overline{\mathbf{Y}})$
X^Y for $0 < X$ or $0 < Y$	$\mathbf{min}(\underline{\mathbf{X}}^{\underline{\mathbf{Y}}}, \underline{\mathbf{X}}^{\overline{\mathbf{Y}}}, \overline{\mathbf{X}}^{\underline{\mathbf{Y}}}, \overline{\mathbf{X}}^{\overline{\mathbf{Y}}})$	$\mathbf{max}(\underline{\mathbf{X}}^{\underline{\mathbf{Y}}}, \underline{\mathbf{X}}^{\overline{\mathbf{Y}}}, \overline{\mathbf{X}}^{\underline{\mathbf{Y}}}, \overline{\mathbf{X}}^{\overline{\mathbf{Y}}})$
$\min(X, Y)$	$\mathbf{min}(\underline{\mathbf{X}}, \underline{\mathbf{Y}})$	$\mathbf{min}(\overline{\mathbf{X}}, \overline{\mathbf{Y}})$
$\max(X, Y)$	$\mathbf{max}(\underline{\mathbf{X}}, \underline{\mathbf{Y}})$	$\mathbf{max}(\overline{\mathbf{X}}, \overline{\mathbf{Y}})$

Arithmetic with independence

	Mean	*Variance*
$X+Y$	$\mathbf{EX}+\mathbf{EY}$	$\mathbf{VX}+\mathbf{VY}$
$X-Y$	$\mathbf{EX}-\mathbf{EY}$	$\mathbf{VX}+\mathbf{VY}$
$X \times Y$	$\mathbf{EXEY}$	$(\mathbf{EX})^2\mathbf{VY}+(\mathbf{EY})^2\mathbf{VX}+\mathbf{VXVY}$
$\frac{X}{Y}$ for $0 \notin Y$	$E(X \times (1/Y))$	$V(X \times (1/Y))$
X^Y for $0 < X$ or $0 < Y$	$E(\exp(\ln(X) \times Y))$	$V(\exp(\ln(X) \times Y))$
$\max(X, Y)$	$\begin{cases}\mathbf{EX}, & \text{if } Y < X \\ \mathbf{EY}, & \text{if } X < Y \\ \text{“Bertsimas max”}, & \text{otherwise}\end{cases}$	$\begin{cases}\mathbf{VX}, & \text{if } Y < X \\ \mathbf{VY}, & \text{if } X < Y \\ \text{env}(\max(VX, VY), 0), & \text{otherwise}\end{cases}$
$\min(X, Y)$	$\begin{cases}\mathbf{EX}, & \text{if } X < Y \\ \mathbf{EY}, & \text{if } Y < X \\ \text{“Bertsimas min”}, & \text{otherwise}\end{cases}$	$\begin{cases}\mathbf{VX}, & \text{if } X < Y \\ \mathbf{VY}, & \text{if } Y < X \\ \text{env}(\max(VX, VY), 0), & \text{otherwise}\end{cases}$

Arithmetic without dependence assumptions (Fréchet)

	Mean	*Variance*
$k+X$ (shifting)	$\mathbf{k}+\mathbf{EX}$	$\mathbf{VX}$
kX (rescaling)	$\mathbf{kEX}$	$\mathbf{k}^2\mathbf{VX}$
e^X	rowe(exp)	rowevar(exp)
$\ln(X)$ for $0 < X$	rowe(ln)	rowevar(ln)
$\log_{10}(X)$ for $0 < X$	rowe($\log_{10}$)	rowevar($\log_{10}$)
$\frac{1}{X}$ for $0 \notin X$	rowe(reciprocal)	rowevar(reciprocal)
X^2	$(\mathbf{EX})^2+\mathbf{VX}$	rowevar(square)
$\lvert X\rvert$ (absolute value)	$\begin{cases}\mathbf{EX}, & \text{if } 0 \le \underline{X} \\ -\mathbf{EX}, & \text{if } \overline{X} \le 0 \\ [\lvert EX\rvert, \lvert EX\rvert + \sqrt{VX}(\pi - \text{atan}(\frac{\lvert EX\rvert}{\sqrt{VX}}))], & \text{if } 0 \in X\end{cases}$	$\max(0, EX^2 + VX - E(\lvert X\rvert)^2)$
$\sqrt{X}$ for $0 \le X$	rowe($\sqrt{}$)	rowevar($\sqrt{}$)
$X+Y$	$\mathbf{EX}+\mathbf{EY}$	$(\sqrt{\mathbf{VX}} \pm \sqrt{\mathbf{VX}})^2$
$X-Y$	$\mathbf{EX}-\mathbf{EY}$	$(\sqrt{\mathbf{VX}} \pm \sqrt{\mathbf{VX}})^2$
$X \times Y$	$\mathbf{EXEY} \pm \sqrt{\mathbf{VXVY}}$	“Goodman”
$\frac{X}{Y}$ for $0 \notin Y$	$E(X \times (1/Y))$	$V(X \times (1/Y))$
X^Y for $0 < X$ or $0 < Y$	$E(\exp(\ln(X) \times Y))$	$V(\exp(\ln(X) \times Y))$
$\max(X, Y)$	“Bertsimas max”	$\text{env}(\max(VX, VY), 0)$
$\min(X, Y)$	“Bertsimas min”	$\text{env}(\max(VX, VY), 0)$

University of Strathclyde Glasgow

Compute with missing information

Mean and variance bounds can be estimated from range

```
julia> A = Moments(mean = missing, var = missing, range = interval(2.3, 7))
moment:              ~ ( mean = [2.3,7.0], var = [0.0,5.5225] , range = [2.3,7.0] )

julia> B = Moments(mean = 3, var = missing, range = interval(2.3, 7))
moment:              ~ ( mean  = 3, var = [0.0,2.8000000000000007] , range = [2.3,7.0] )

julia> C = Moments(mean = interval(5, 10), var = missing, range = interval(2.3, 7))
moment:              ~ ( mean = [5.0,7.0], var = [0.0,5.4] , range = [2.3,7.0] )

julia> D = Moments(mean = interval(10, 30), var = missing, range = interval(2.3, 7))
ERROR: ArgumentError: Provided information not valid. Mean ∩ Range = ∅.
       [10, 30] ∩ [2.29999, 7] = ∅

julia> E = Moments(mean = 3, var = interval(15,18), range = interval(2.3, 7))
ERROR: ArgumentError: Provided information not valid. Variance ∩ VarBounds = ∅.
       [15, 18] ∩ [0, 2.80001] = ∅
```

Mean and variance bounded from ranges

Tighter variance when mean is given

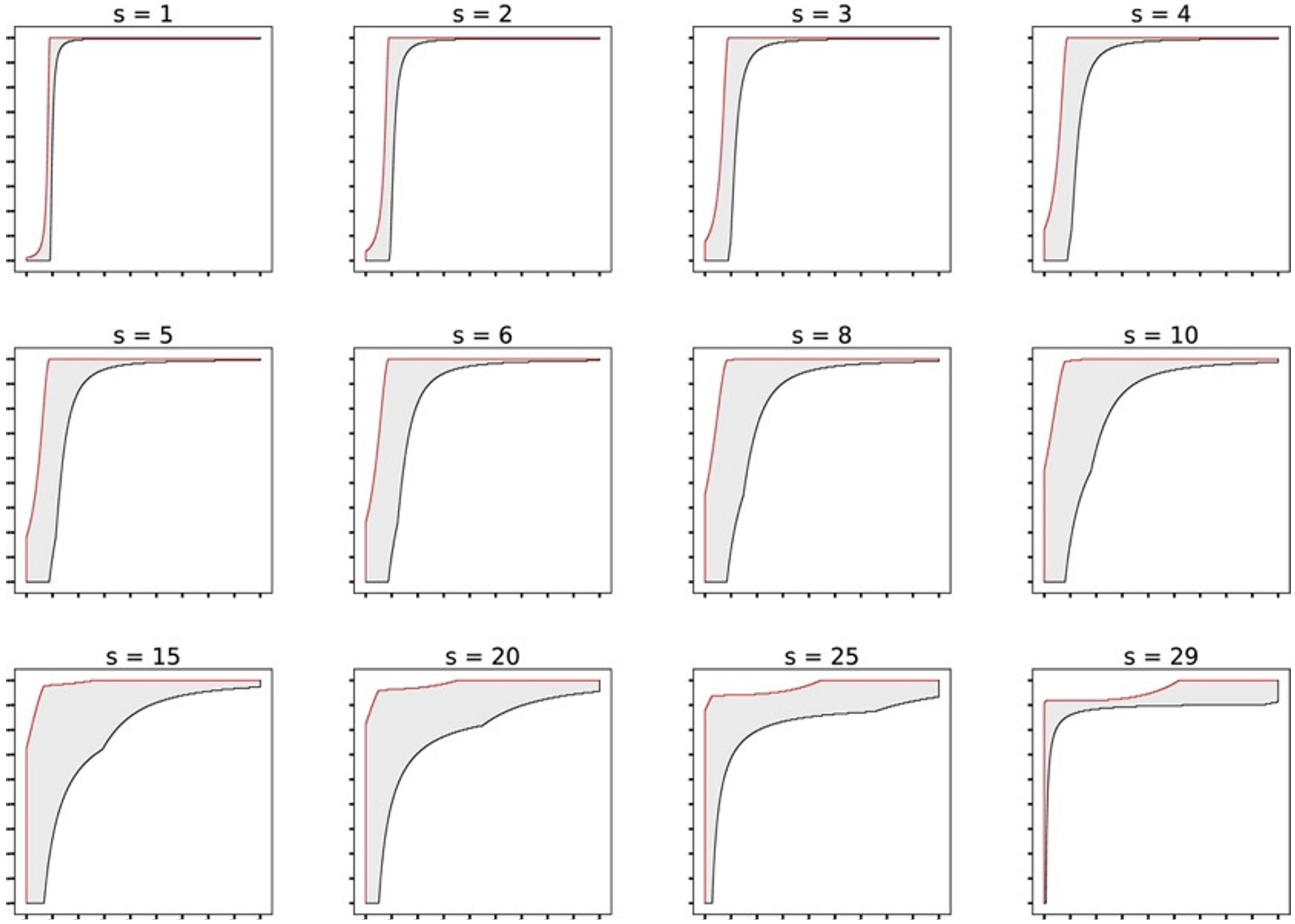
Given mean is too wide

Inconsistent mean

Inconsistent variance

Bounds of CDF

$$[\underline{X}, \overline{X}] = [0, 100], EX = 10, VX = s^2$$



Arithmetic tables with intervals

- Even starting with point estimates from means and variances, moment arithments generally yields interval results
- EX, VX and bounds can be intervals as well

Repeated variables problem:

Can be addressed using significance arithmetic, affine arithmetic, Taylor models and relation arithmetic

Simple numerical solution: **Subintervalisation**

Interval is split into *n* (usually linearly spaced) sub-intervals, and the expression is evaluated *n* times with each sub-interval.

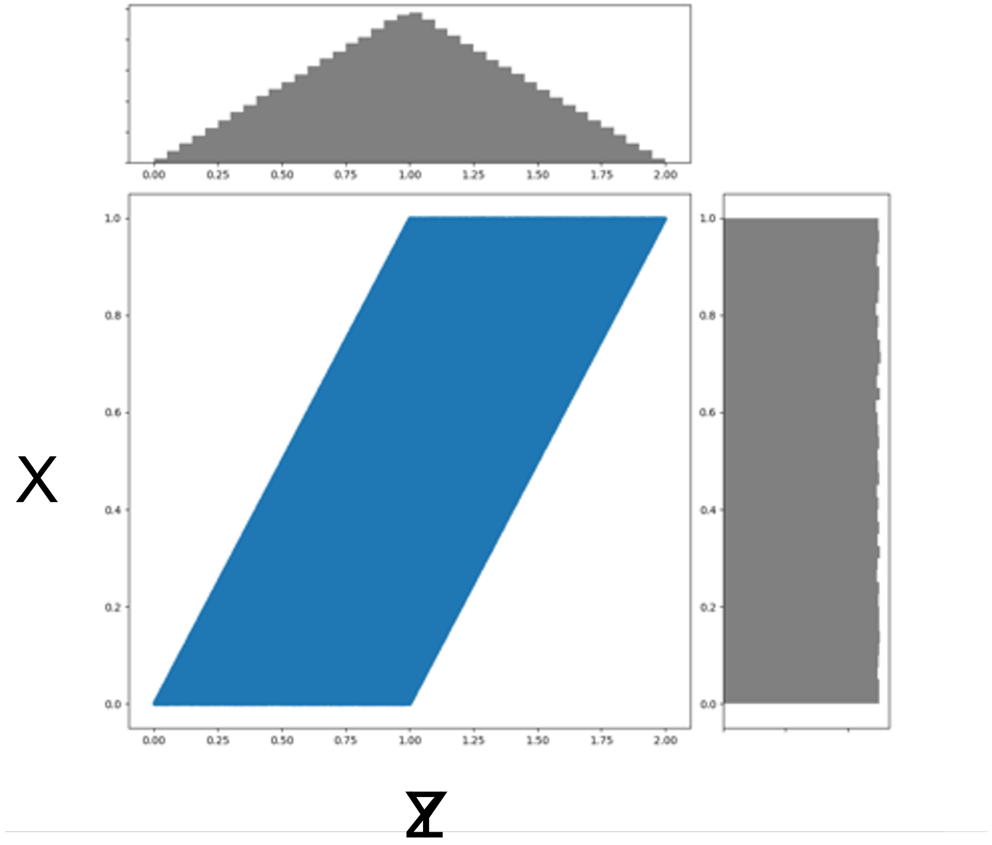
- n^m interval calculations
- Feasible since only 2 variable are intervals
- Generally $2^{15} = 32768$

Dependent variables

$$Z = X + Y \quad \Big| \quad [-1,1] + [-1,1] = [-2,2]$$

$$Y \subset \tilde{Y} \qquad [-1,1] \subset [-3,3]$$

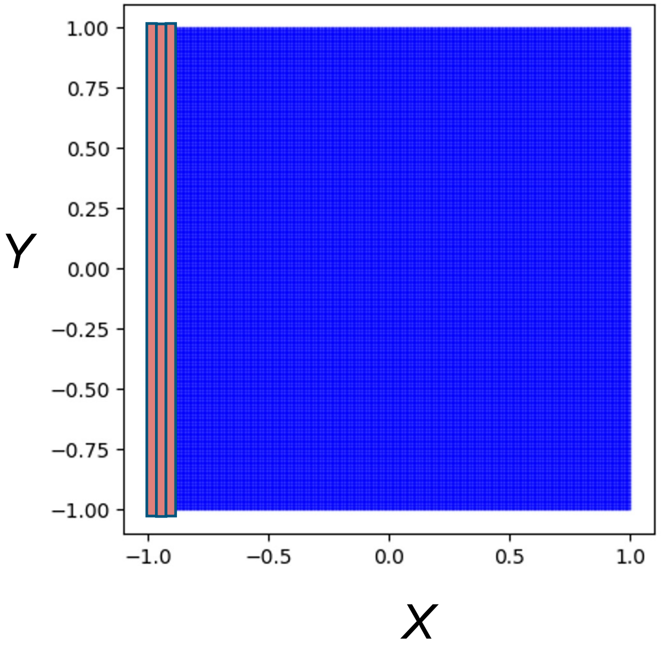
Even if *X* and *Y* are non-dependent,
Z and *X* are dependent

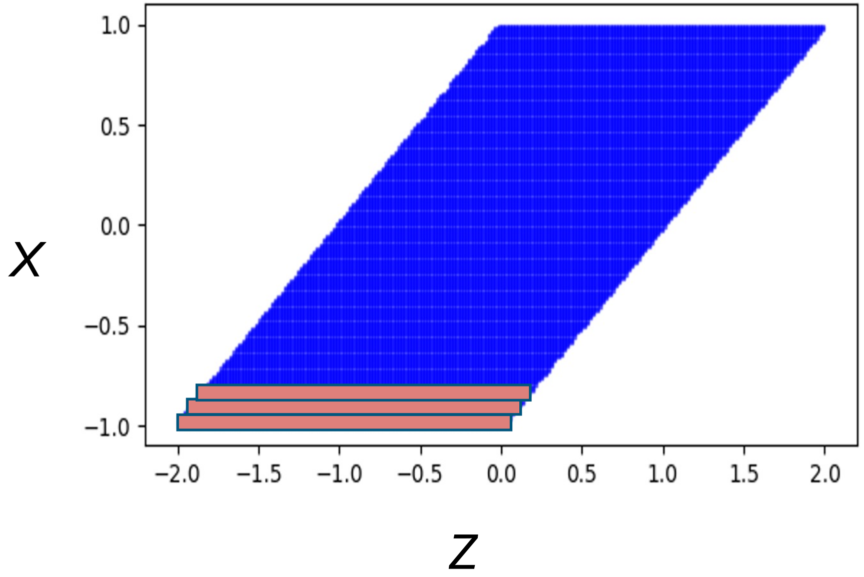


Zone arithmetic

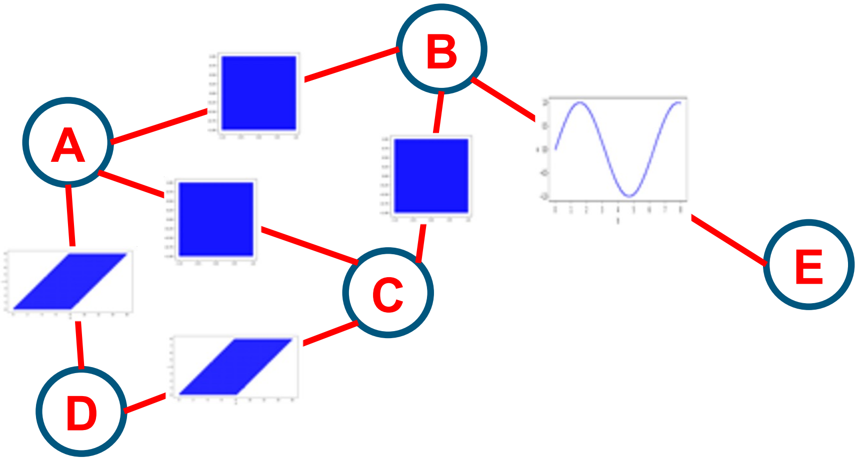
https://github.com/AnderGray/ZoneArithmetic.jl

$$Z = X + Y$$





Tracking dependencies

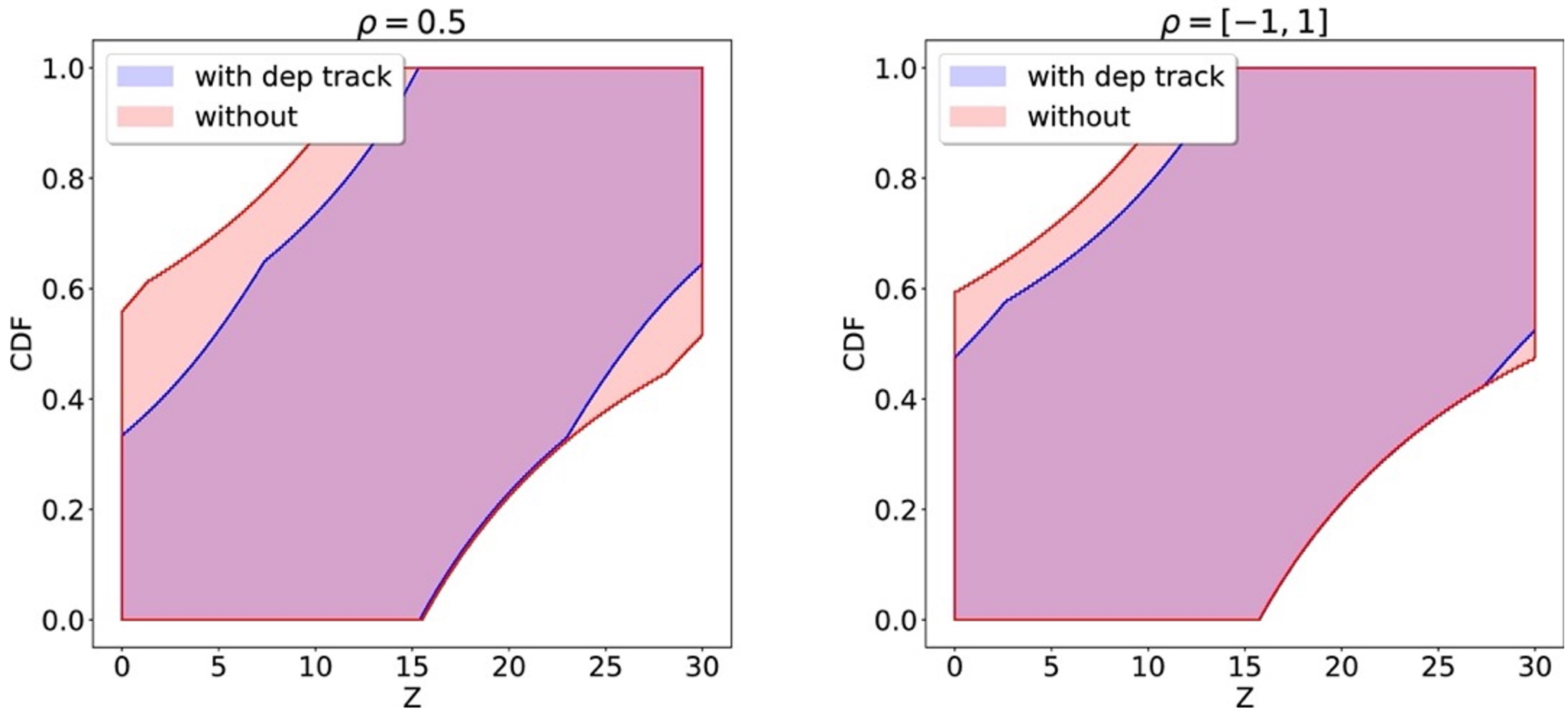


- Dependencies can be tracked in a graph
- The nodes are variables (intervals)
- The edges are dependencies (zones)
- When a new variable is calculated, it's added and linked in the graph

```
julia> d = a + c
intervalZ([0, 16], 4)
```

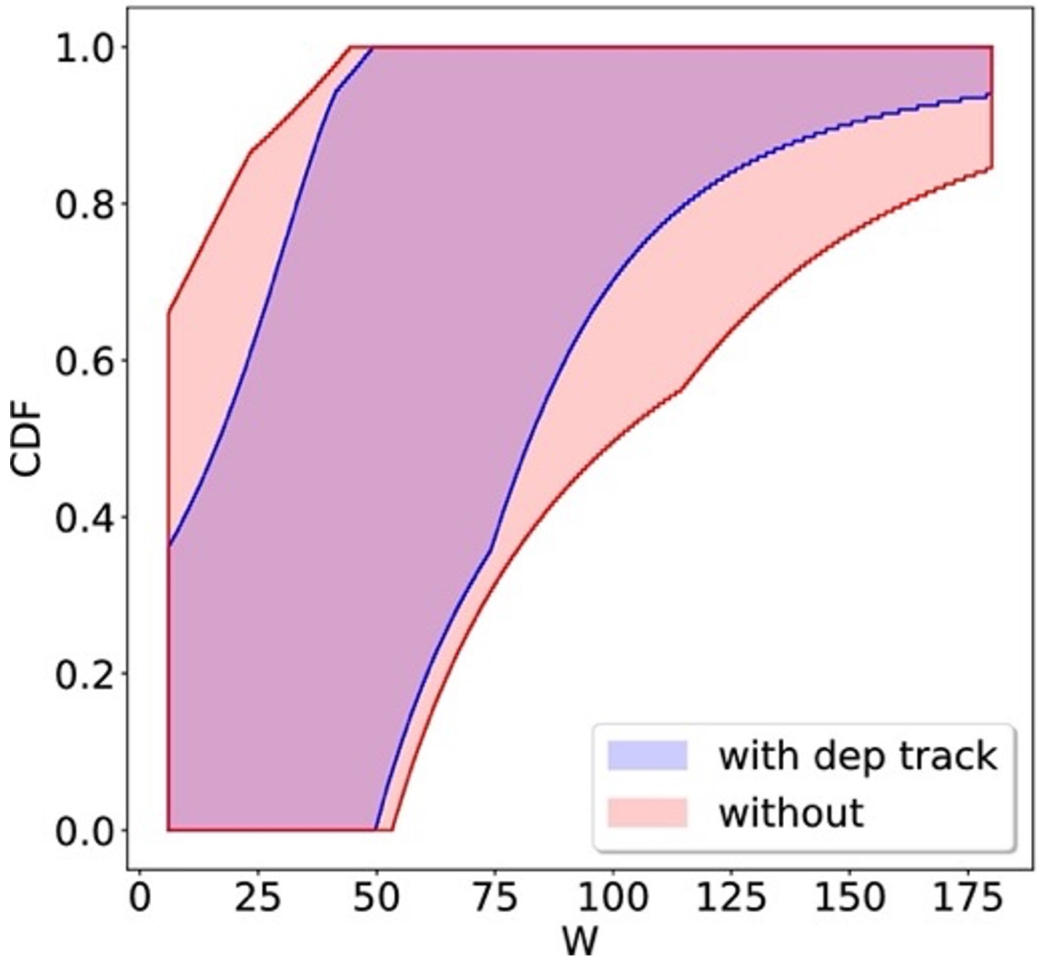
Dependence tracking

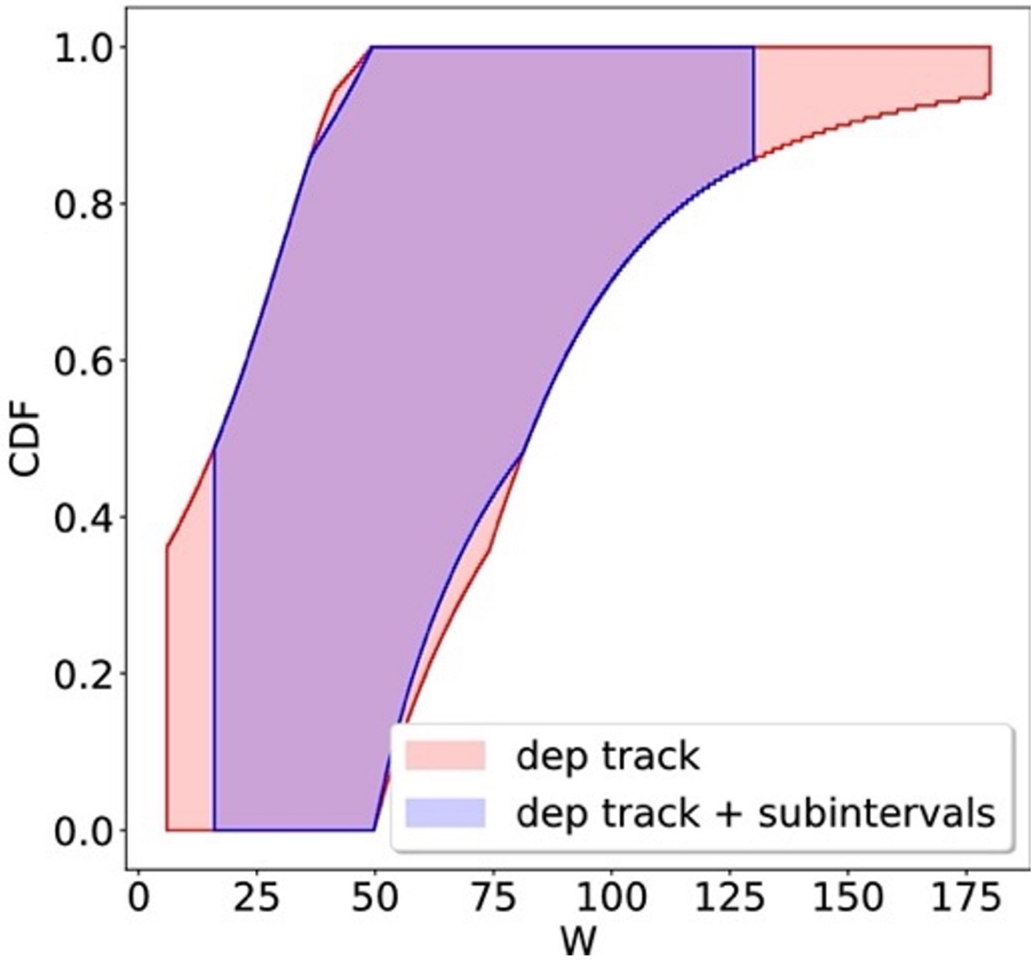
$$Z = (X + Y)X$$



Dependence tracking with subintervals

$$W = (X + Y)(Z - Y)$$

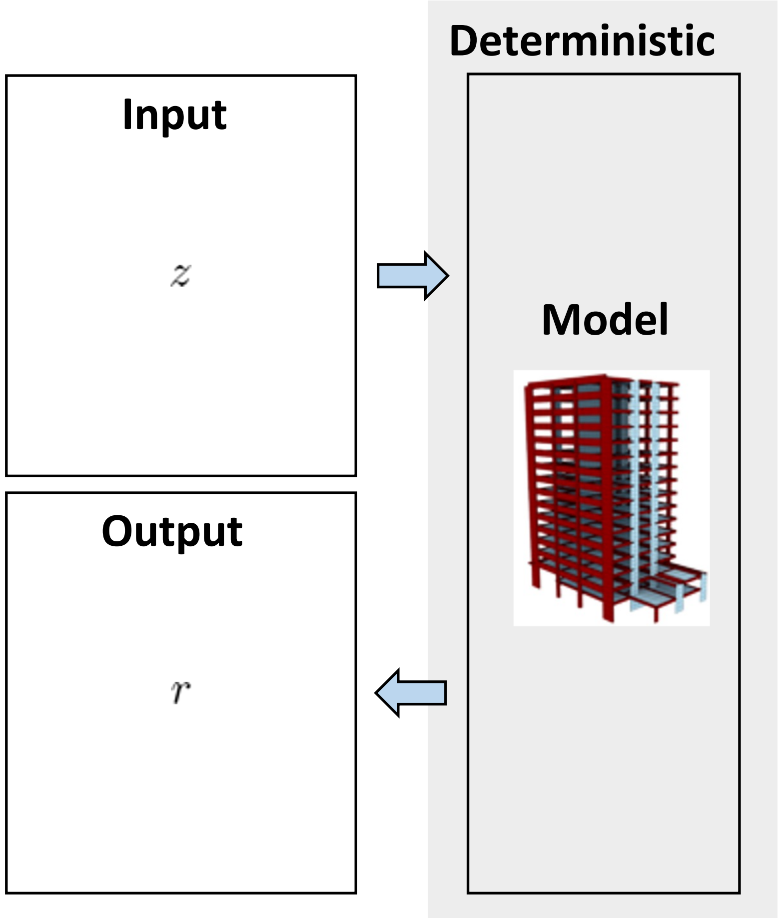




How do we make computations with imprecise probabilities in numerical models?

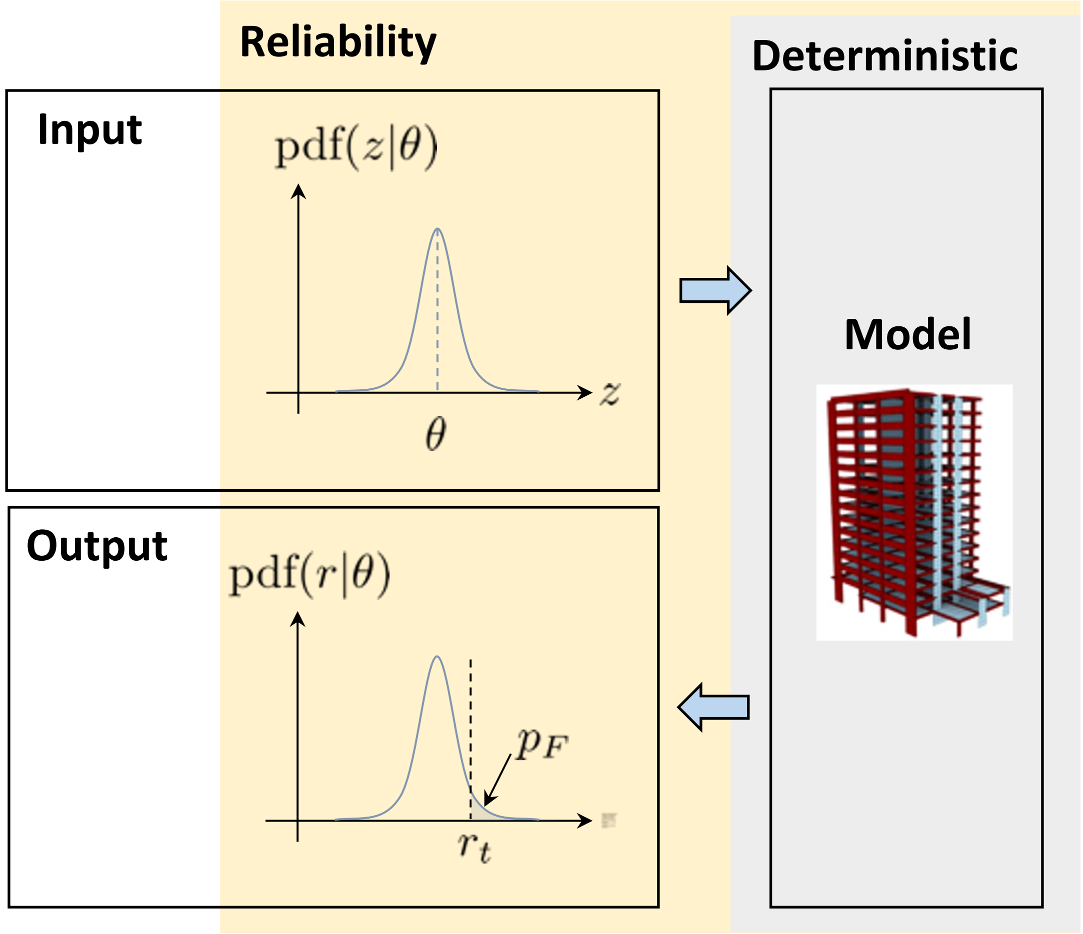
Motivation – Deterministic Analysis

- Engineering system represented by numerical model (e.g. finite elements)
- Model depends on inputs z
  - model parameters
  - initial / boundary conditions
- Response of interest r
  - displacement / velocity / acceleration
  - stress / strain
  - …



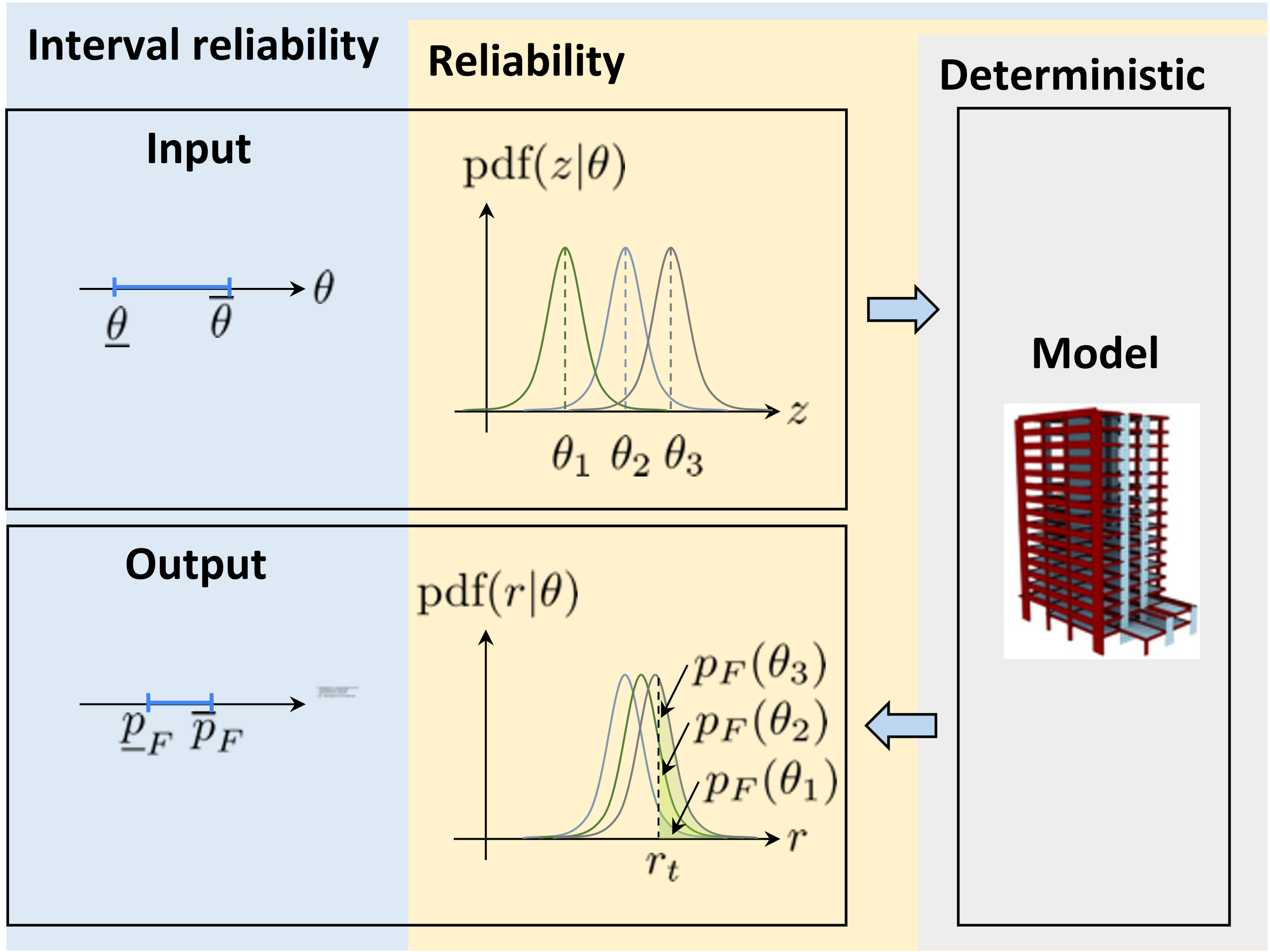
Motivation – Reliability Analysis

- Aleatory uncertainty of input modeled with probability distribution, depends on parameter θ (e.g. mean)
- Response must not exceed threshold r_t
- Probability of undesirable behavior: p_F



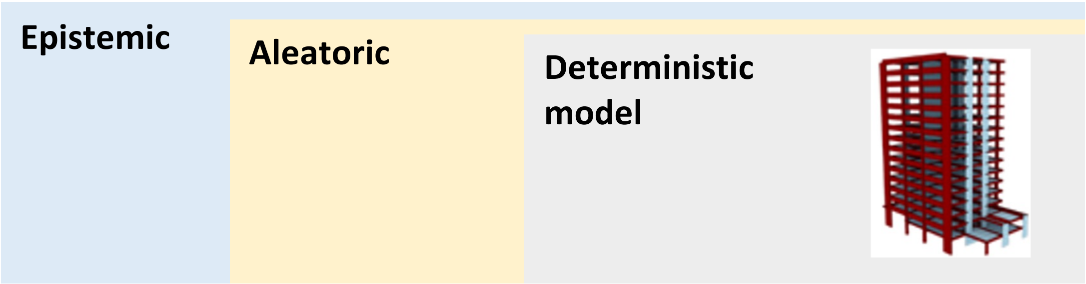
Motivation – Interval Reliability Analysis

- Epistemic uncertainty on θ modeled as interval (parametric p-box)
- p_F belongs to an interval



Decoupling approaches

- Coping with aleatoric and epistemic uncertainty: huge challenge!



- What we would like: decoupling of the uncertainty

Introduced approaches

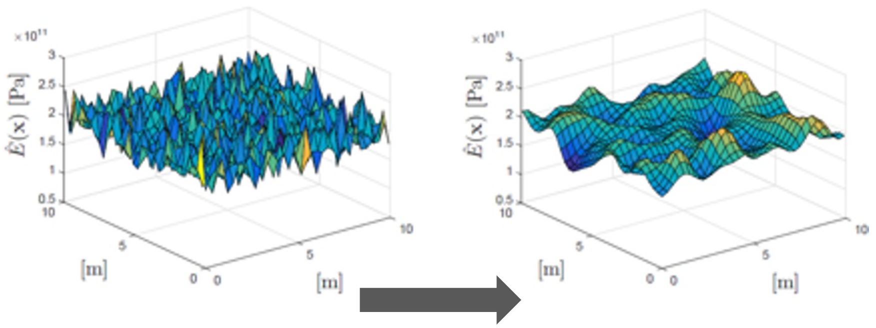
- Decoupling of uncertainty
  - additive relation between $N_{aleatory}$ and $N_{epistemic}$
- Three approaches developed recently:
  - decoupling based on operator norm theory
  - augmented space approaches
  - Bayesian quadrature
- Savings in computational cost with several orders of magnitude

Operator norm framework: scope

- Linear systems subject to epistemic ($\boldsymbol{\theta}_s$) and aleatory ($\boldsymbol{y}$) uncertainties
- Gaussian forces affected subject to ($\boldsymbol{\theta}_f$) and aleatory ($\boldsymbol{z}$) uncertainty

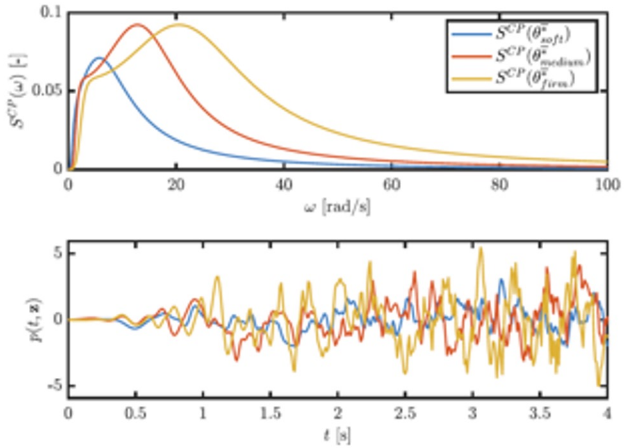
$$\overset{\text{response}}{\boldsymbol{r}(\boldsymbol{\theta},\boldsymbol{y},\boldsymbol{z})} = \overset{\text{system}}{\boxed{\boldsymbol{A}(\boldsymbol{\theta}_s,\boldsymbol{y})}}\overset{\text{force}}{\boxed{\boldsymbol{f}(\boldsymbol{\theta}_f,\boldsymbol{z})}}$$

Imprecise Random field

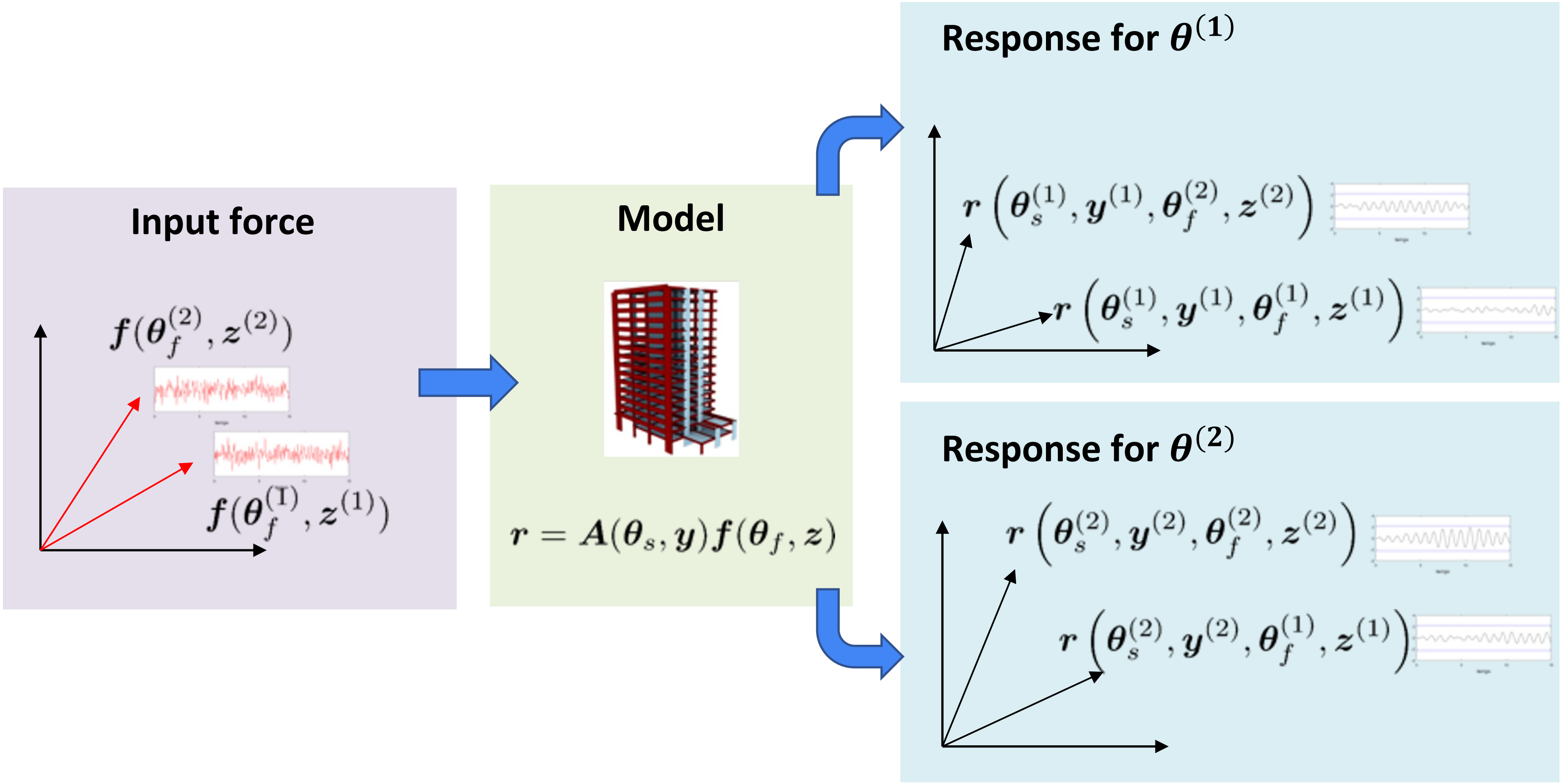


Correlation length increases

Imprecise PSD

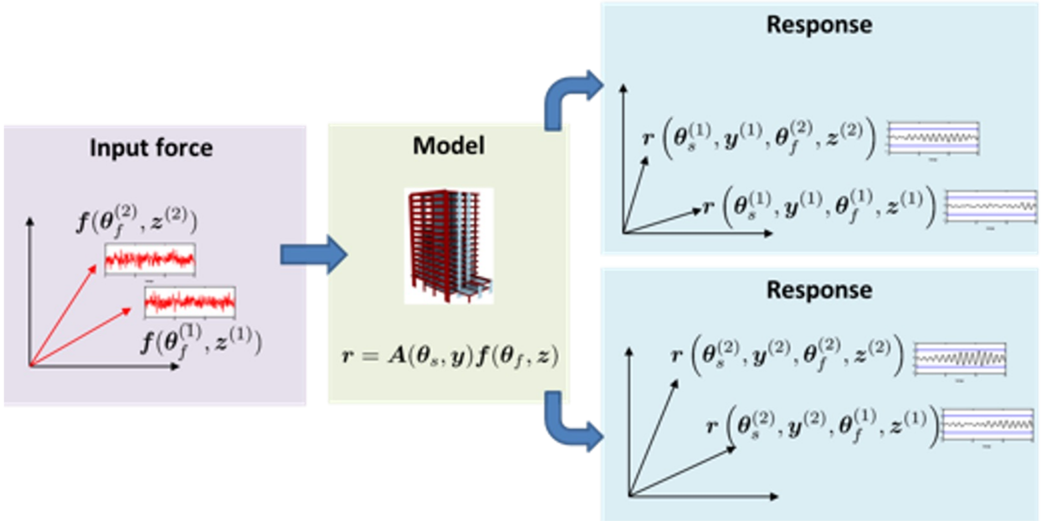


Operator Norm Theorem (1/2)



- Response r is the result of ***stretching*** loading $f(\theta_f, z)$ by $A(\theta_S, y)$
- Less stretching leads to **smaller** p_F; more stretching leads to **higher** p_F

Operator Norm Theorem (2/2)



- The amount of ***stretching*** induced by $\boldsymbol{A}$ can be bounded from above by the **operator norm theorem**

$$||\boldsymbol{A}||_{\infty,2} = \inf\{c \geq 0 : ||\boldsymbol{A}\boldsymbol{v}||_{\infty} \leq c\,||\boldsymbol{v}||_2\}$$

- In this case, operator norm corresponds to **maximum standard deviation of response** $\sigma_{\max}(\boldsymbol{\theta})$

Proposed approach

Optimization (epistemic parameters)

Deterministic model

+

Reliability (aleatory Parameters)

Deterministic model

$$\boxed{\underline{\boldsymbol{\theta}}^*} = \underset{\boldsymbol{\theta}\in[\underline{\boldsymbol{\theta}},\overline{\boldsymbol{\theta}}]}{\operatorname{argmin}} \left(\sigma_{\max}(\boldsymbol{\theta})\right)$$

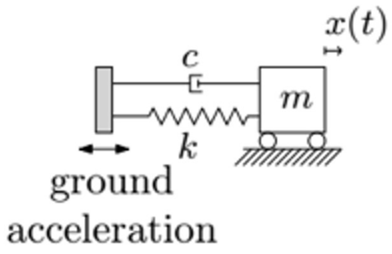
$$\boxed{\overline{\boldsymbol{\theta}}^*} = \underset{\boldsymbol{\theta}\in[\underline{\boldsymbol{\theta}},\overline{\boldsymbol{\theta}}]}{\operatorname{argmax}} \left(\sigma_{\max}(\boldsymbol{\theta})\right)$$

Determined <u>deterministically</u> via linearization
⇝ only required for stochastic system matrices

$\boxed{\underline{p}_F}$ $\boxed{\overline{p}_F}$ $\rightarrow p_F$

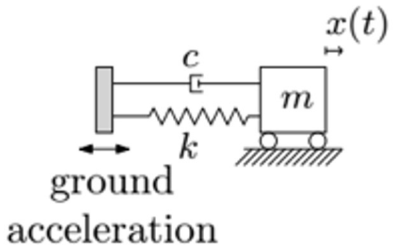
- <u>Proposed approach involves</u>:
  - **Two deterministic optimization** problems
  - **Two reliability** problems

Example 1

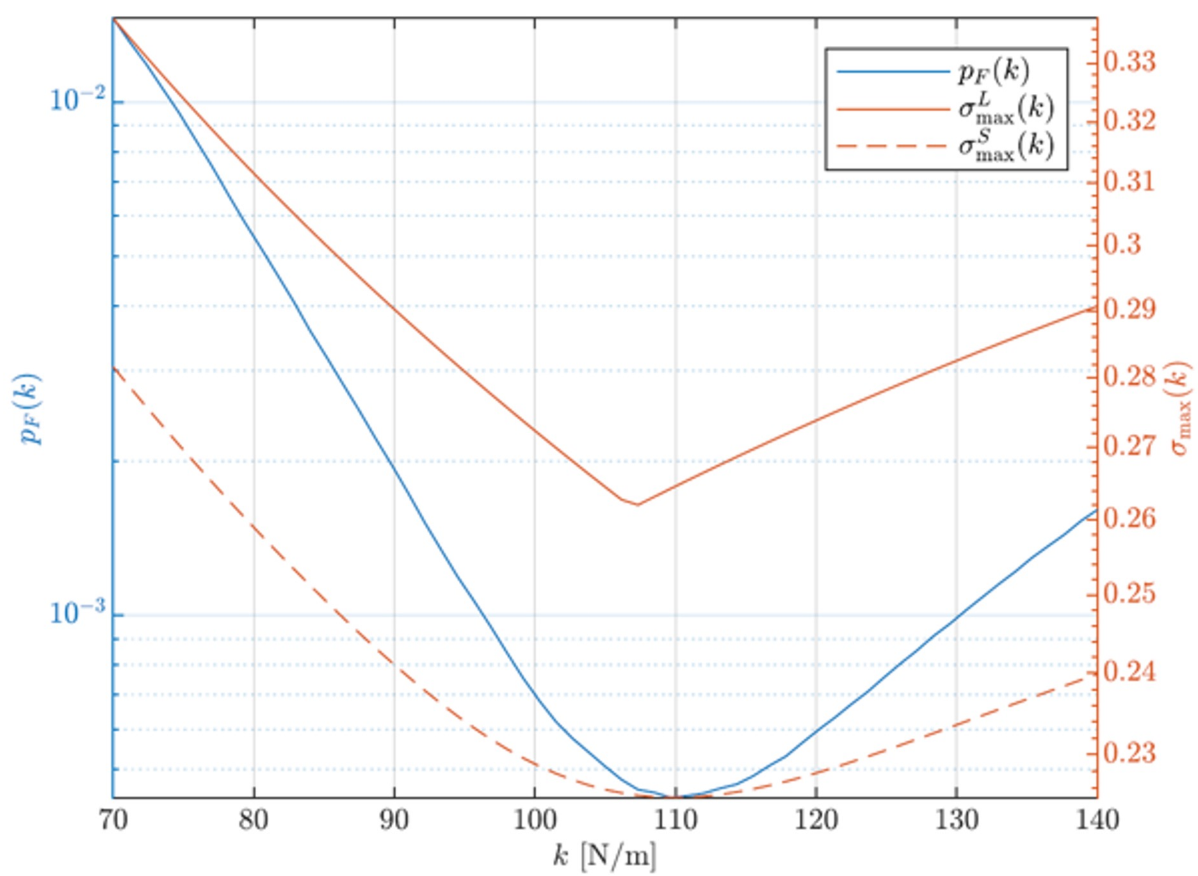


- SDOF oscillator subject to stochastic ground acceleration
  - Aleatoric ground acceleration modeled using Clough-Penzien model
  - Aleatoric mass (lognormal)
  - Epistemic stiffness
  - Failure criterion: first excursion, involves maximum displacement and maximum acceleration

Example 1



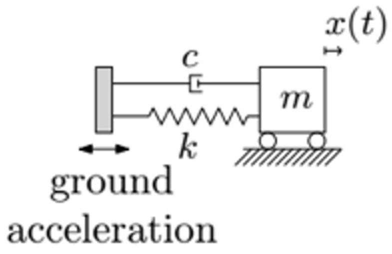
- SDOF oscillator subject to stochastic ground acceleration



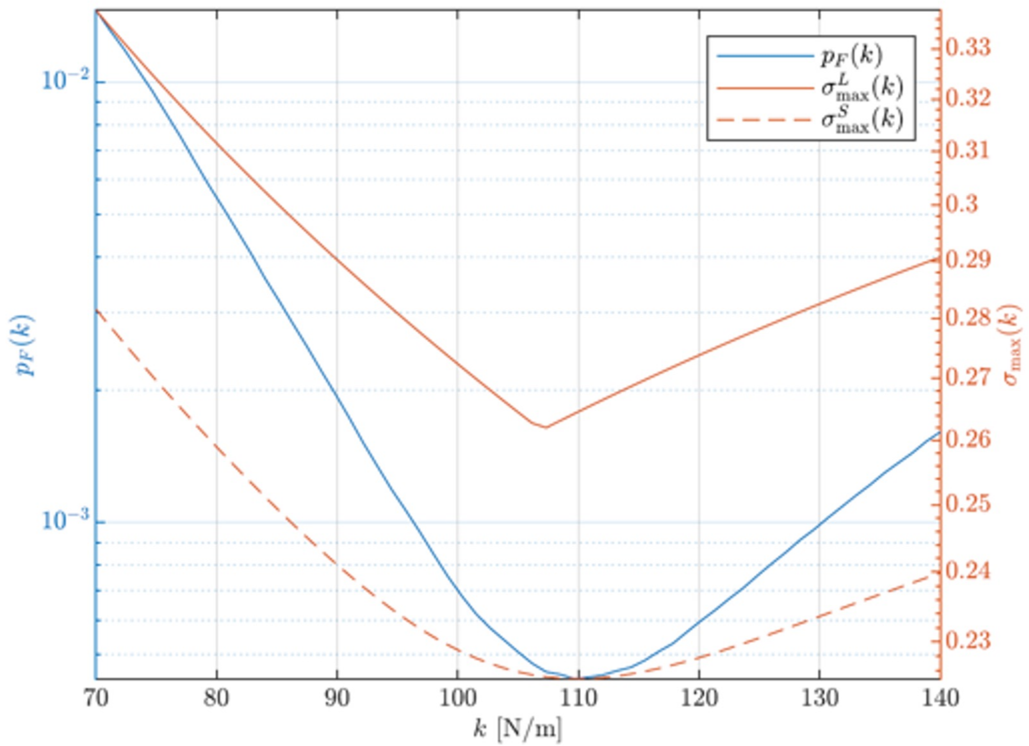
- $\sigma^L_{\max}$: maximum standard deviation estimated by linearization
- $\sigma^S_{\max}$: maximum standard deviation estimated by

Minimum / maximum values of (σ^L_{max}, σ^S_{max}) occur at minimum / maximum values of p_F

Example 1



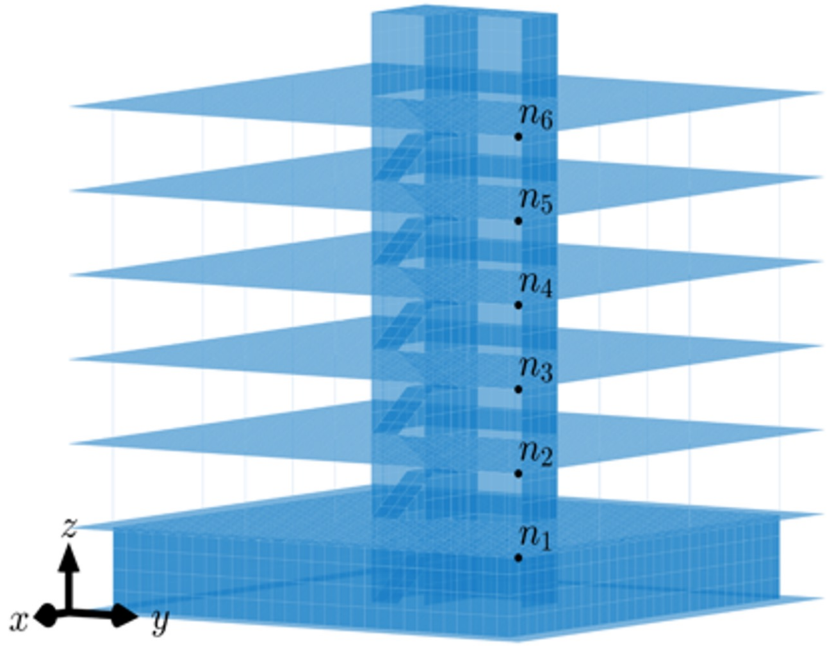
- SDOF oscillator subject to stochastic ground acceleration



	Proposed approach		Direct optimization (DIS)	
	lower bound	upper bound	lower bound	upper bound
p_F	4.6×10^{-3}	1.5×10^{-2}	4.4×10^{-3}	1.5×10^{-2}
k [N/m]	107	70	111	70
Relative execution time	1		64.9	

Example 2: a six-story building

- 6 story building
  - Reinforced concrete
  - 9500 shell & beam elements
- QOI: interstory drift
- Load: earthquake

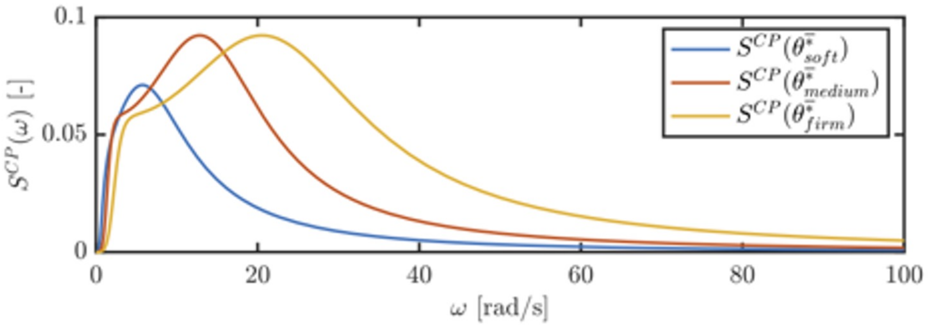


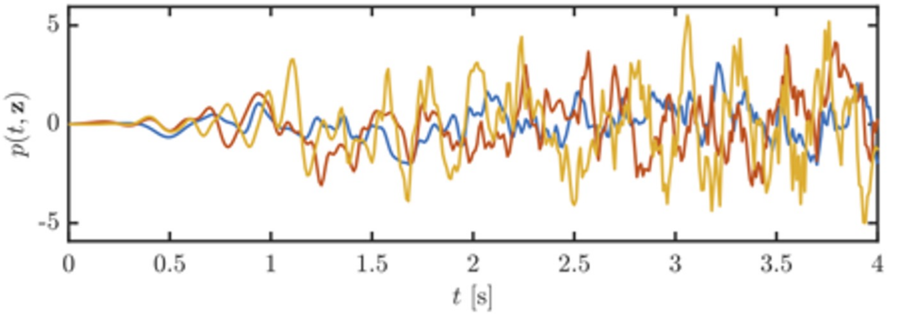
Load acting on the building

Soil type	ω_g [rad/s]	ζ_g	ω_f [rad/s]	ζ_f
Firm	8π	0.60	0.8π	0.60
Medium	5π	0.60	0.5π	0.60
Soft	2.4π	0.85	0.24π	0.85

- Earthquake is modelled as stochastic process:
  - Gaussian stochastic process
  - Autocorrelation governed by modulated Clough-Penzien spectrum:

$$S_{CP}(\omega) = \frac{\omega_g^4 + (2\zeta_g\omega_g\omega)^2}{(\omega_g^2-\omega^2)^2 + (2\zeta_g\omega_g\omega)^2} \cdot \frac{\omega^4}{(\omega_f^2-\omega^2)^2 + (2\zeta_g\omega_g\omega)^2} \cdot S_0$$



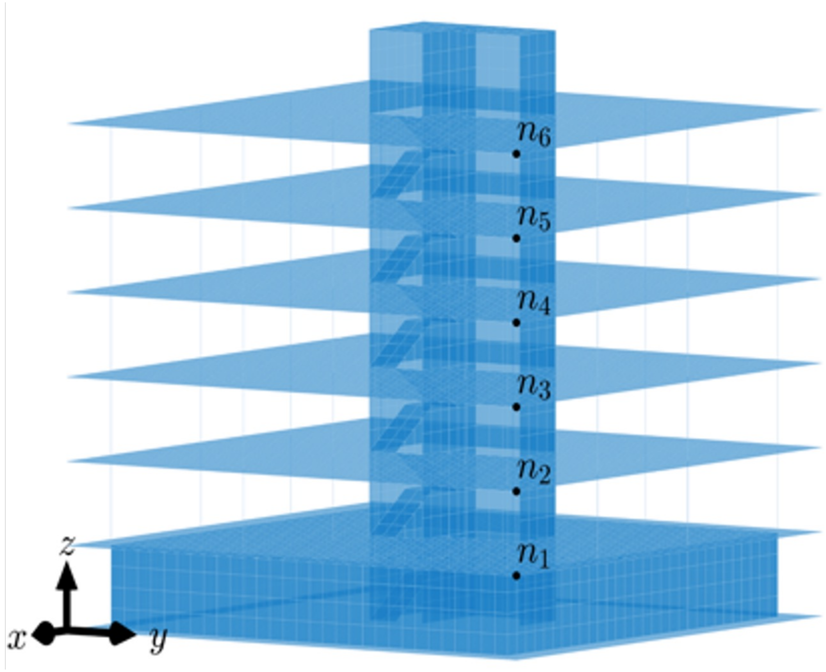


Uncertainty model

- Clough-Penzien spectrum:

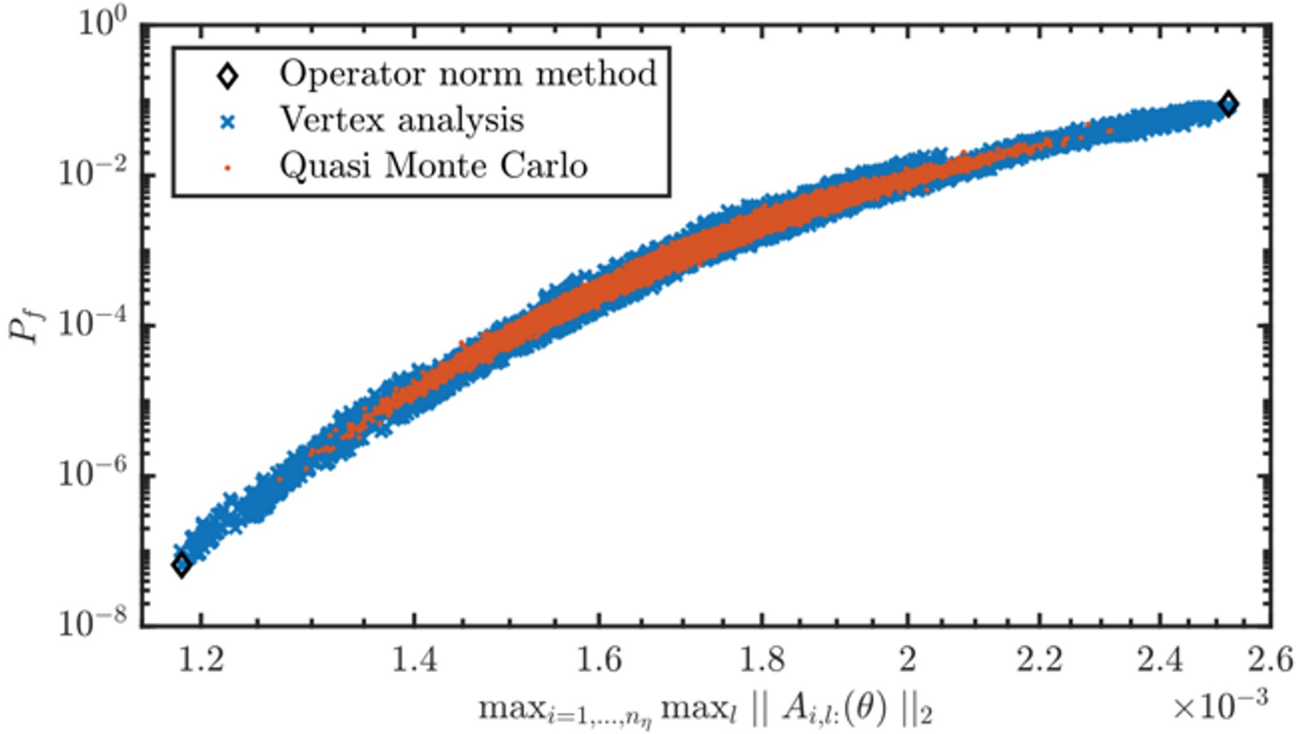
ω_g^I	ω_f^I	ζ_g^I	ζ_f^I	S_0^I	c_1^I	c_2^I
$\pi[2.4; 8]$	$\pi[0.24; 0.8]$	$[0.6; 0.85]$	$[0.6; 0.85]$	$4 \times 10^{-2}[0.75; 1.25]$	$[0.12; 0.16]$	$[0.14; 0.18]$

- Gaussian random process:
  - 1300 random variables
- Concrete material of each floor:
  - 6 Young's moduli: $E^I = [2.07; 2.53]\times 10^{+10}$



Example 2: results

- Optima in operator norm correspond to optima in failure probability
- Large reduction in computational cost:
  - Double loop MCS:
    5.000.000 FE simulations
  - Vertex analysis:
    4.096.000 FE simulations
  - Operator norm:
    3500 FE simulations



Extension of the method to nonlinear dynamics

- Main question: How to apply this method to systems of following form?

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t,\boldsymbol{z}) + \boldsymbol{C}\dot{\boldsymbol{x}}(t,\boldsymbol{z}) + \boldsymbol{K}\boldsymbol{x}(t,\boldsymbol{z}) + \boldsymbol{\Phi}(\ddot{\boldsymbol{x}},\dot{\boldsymbol{x}},\boldsymbol{x}) = \boldsymbol{\rho p}(t,\boldsymbol{z})$$

- Idea: Define a linearized system of equations as:

$$(\boldsymbol{M}+\boldsymbol{M}_e)\ddot{\boldsymbol{x}}(t,\boldsymbol{z}) + (\boldsymbol{C}+\boldsymbol{C}_e)\dot{\boldsymbol{x}}(t,\boldsymbol{z}) + (\boldsymbol{K}+\boldsymbol{K}_e)\boldsymbol{x}(t,\boldsymbol{z}) = \boldsymbol{\rho p}(t,\boldsymbol{z})$$

- Where $\boldsymbol{M}_e, \boldsymbol{C}_e$ and $\boldsymbol{K}_e$ are determined by minimizing

$$\epsilon = \boldsymbol{\Phi}(\ddot{\boldsymbol{x}},\dot{\boldsymbol{x}},\boldsymbol{x}) - \boldsymbol{M}_e\ddot{\boldsymbol{x}}(t,\boldsymbol{z}) + \boldsymbol{C}_e\dot{\boldsymbol{x}}(t,\boldsymbol{z}) + \boldsymbol{K}_e\boldsymbol{x}(t,\boldsymbol{z})$$

- Define the operator norm over the system

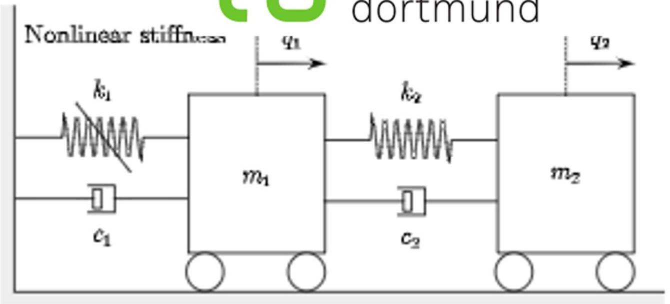
$$\boldsymbol{M}^*\ddot{x}(t,\boldsymbol{z}) + \boldsymbol{K}^*x(t,\boldsymbol{z}) = \boldsymbol{P}^*(t,\boldsymbol{z})$$

- With

$$M^* = \begin{bmatrix} 0 & \boldsymbol{M}+\boldsymbol{M}_e \\ \boldsymbol{M}+\boldsymbol{M}_e & \boldsymbol{C}+\boldsymbol{C}_e \end{bmatrix}, \qquad \boldsymbol{K}^* = \begin{bmatrix} -(\boldsymbol{M}+\boldsymbol{M}_e) & 0 \\ 0 & \boldsymbol{K}+\boldsymbol{K}_e \end{bmatrix}, \qquad \boldsymbol{P}^* = \begin{bmatrix} 0 \\ \boldsymbol{\rho p}(t,\boldsymbol{z}) \end{bmatrix}$$

- Then, identify those parameters $\boldsymbol{\theta}^*$ of a parametric p-box that yield extrema in $\mathcal{P}$
- Compute bounds on P_f using full-scale nonlinear model

Example 1:



- 2 DOF oscillator
- P_1 is a Gaussian stochastic process with Clough-Penzien spectrum
- k_1 is a Duffing-type nonlinearity
- P_f is computed using classical Monte Carlo simulation

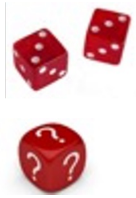
ω_g^I	ω_f^I	ζ_g^I	ζ_f^I	S_0^I
$[0.8, 1.2] \times 4\pi$	$[0.8, 1.2] \times 0.4\pi$	$[0.8, 1.2] \times 0.7$	$[0.8, 1.2] \times 0.7$	$[0.8, 1.2] \times 3 \times 10^{-4}$

parameter	$\underline{P}_f$ (DL)	$\underline{P}_f$ (ON)	$\overline{P}_f$ (DL)	$\overline{P}_f$ (ON)
S_0^*	$2.504 \cdot 10^{-04}$	$2.4087 \cdot 10^{-04}$	$3 \cdot 10^{-04}$	$3.591 \cdot 10^{-04}$
ω_g^*	13.31	15.07	10.31	10.05
ω_f^*	1.506	1.507	1.008	1.005
ζ_g^*	0.825	0.839	0.578	0.5600
ζ_f^*	0.699	0.839	0.637	0.5600
P_f	0.2880	0.2980	0.9800	0.9830
ON	0.0052	0.0048	0.0176	0.0221
n^0	331000	530	226000	520

Literature on the operator norm framework

- M. G. R. Faes and M. A. Valdebenito, ‘Fully decoupled reliability-based optimization of linear structures subject to Gaussian dynamic loading considering discrete design variables’, *Mechanical Systems and Signal Processing*, vol. 156, p. 107616, Jul. 2021, doi: 10.1016/j.ymssp.2021.107616.
- M. G. R. Faes, M. A. Valdebenito, D. Moens, and M. Beer, ‘Operator norm theory as an efficient tool to propagate hybrid uncertainties and calculate imprecise probabilities’, *Mechanical Systems and Signal Processing*, vol. 152, p. 107482, May 2021, doi: 10.1016/j.ymssp.2020.107482.
- M. G. R. Faes and M. A. Valdebenito, ‘Fully decoupled reliability-based design optimization of structural systems subject to uncertain loads’, *Computer Methods in Applied Mechanics and Engineering*, vol. 371, p. 113313, Nov. 2020, doi: 10.1016/j.cma.2020.113313.
- M. G. R. Faes, M. A. Valdebenito, D. Moens, and M. Beer, ‘Bounding the first excursion probability of linear structures subjected to imprecise stochastic loading’, *Computers & Structures*, vol. 239, p. 106320, Oct. 2020, doi: 10.1016/j.compstruc.2020.106320.
- P. Ni, D. J. Jerez, V. C. Fragkoulis, M. G. R. Faes, M. A. Valdebenito, and M. Beer, ‘Operator Norm-Based Statistical Linearization to Bound the First Excursion Probability of Nonlinear Structures Subjected to Imprecise Stochastic Loading’, *ASCE-ASME J. Risk Uncertainty Eng. Syst., Part A: Civ. Eng.*, vol. 8, no. 1, p. 04021086, Mar. 2022, doi: 10.1061/AJRUA6.0001217.

Augmented space methods

y → 

θ → (interval)

- Imprecise probability

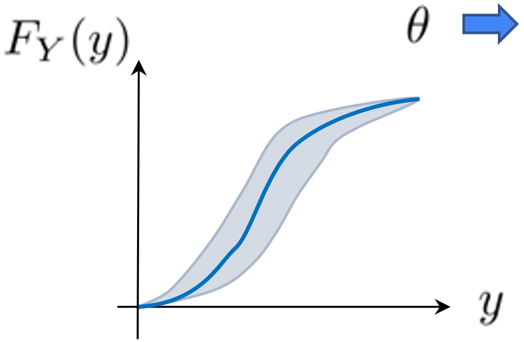
classical probability

$$p_F(\theta) = \int_y I_F(y) f_Y(y|\theta) dy$$

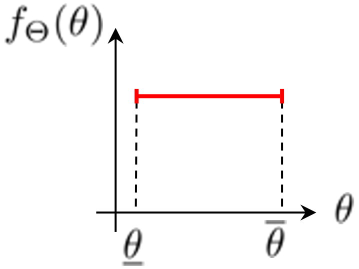
min max

interval probability

$\underline{p}_F$ $\overline{p}_F$

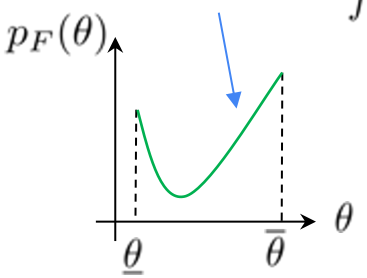


- Augmented reliability problem



$$p_A = \int_\theta \int_y I_F(y) f_Y(y|\theta) f_\Theta(\theta) dy d\theta$$

$$p_F(\theta) = \frac{f_\Theta(\theta|F) p_A}{f_\Theta(\theta)}$$



Model θ as aleatoric **(!)**

Augmented probability

Retrieve $p_F(\theta)$ with **Bayes**

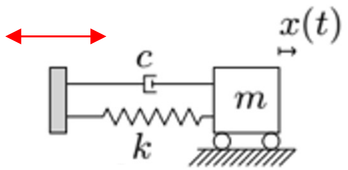
artefact, extracts dependence of p_F w.r.t. θ

technische universität dortmund

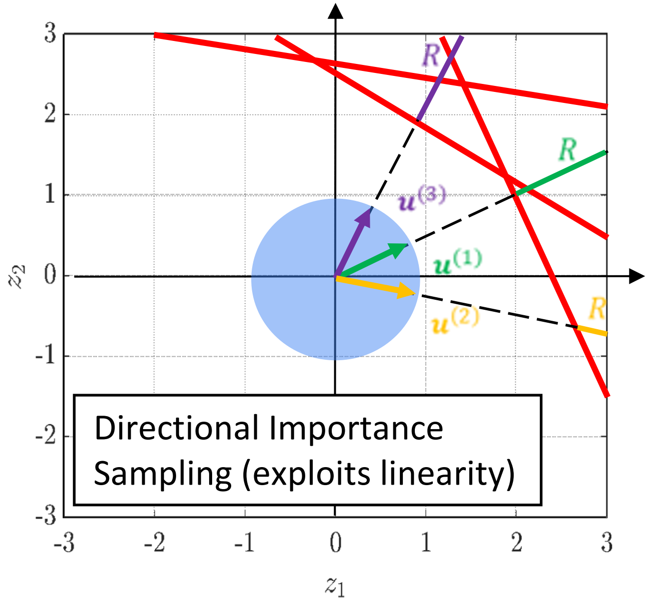
Application: Imprecise reliability in linear dynamics

Stochastic ground acceleration

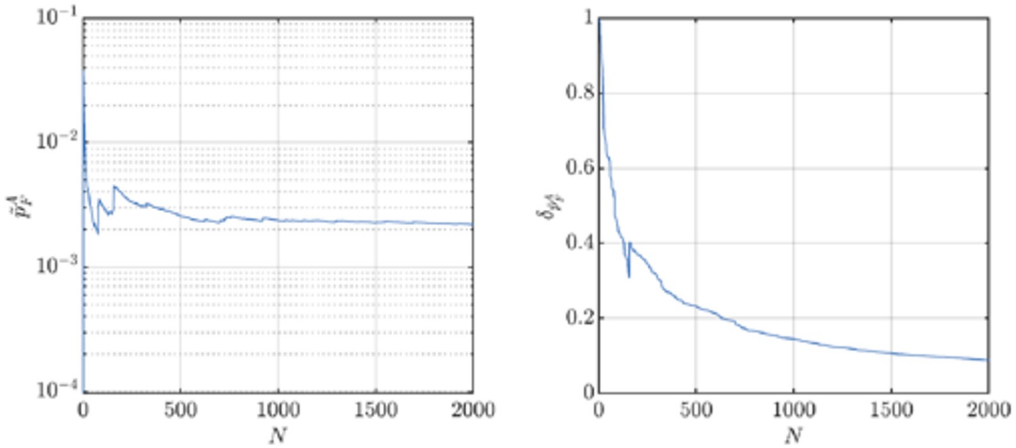
Failure:
- Maximum displacement
- Maximum acceleration

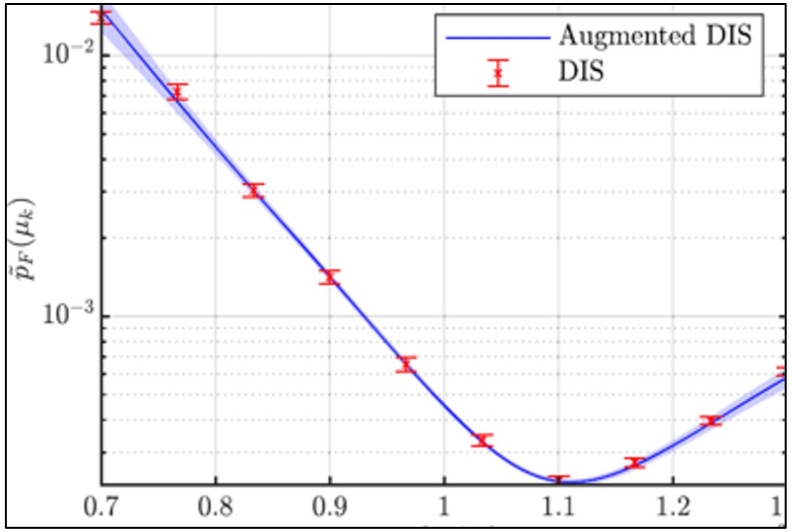


stiffness: parametric p-box (w.r.t. μ_k)



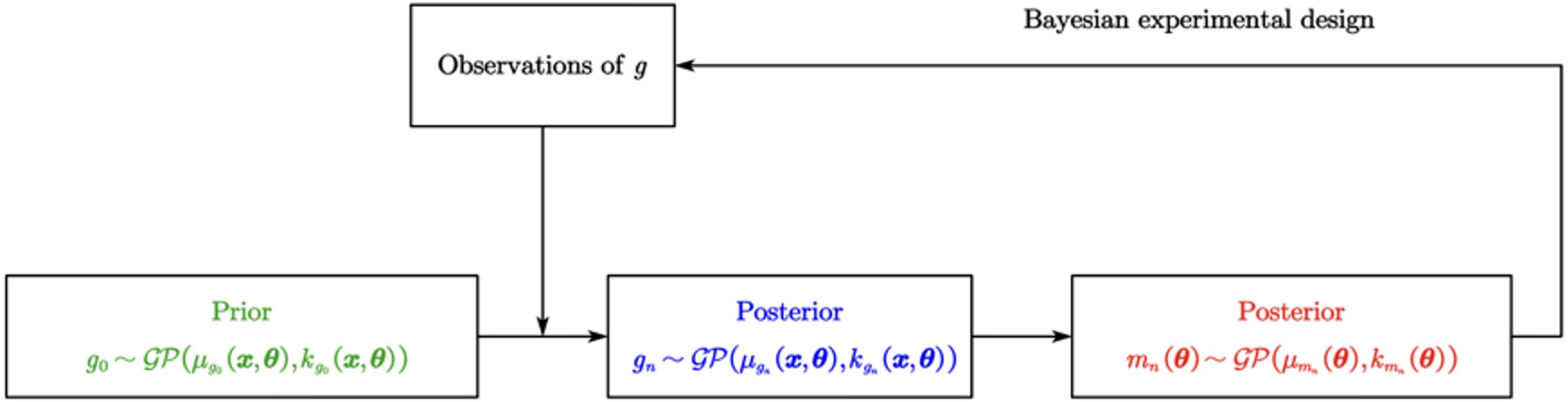
Directional Importance Sampling (exploits linearity)





- Wide range of p_F
- Non-monotonic behaviour captured!

Parallel Bayesian quadrature optimization



$$m(\boldsymbol{\theta}) = \int_{\boldsymbol{\theta} \in [\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}]} g(\boldsymbol{x}) f(\boldsymbol{x}|\boldsymbol{\theta}) \mathrm{d}\boldsymbol{x}$$

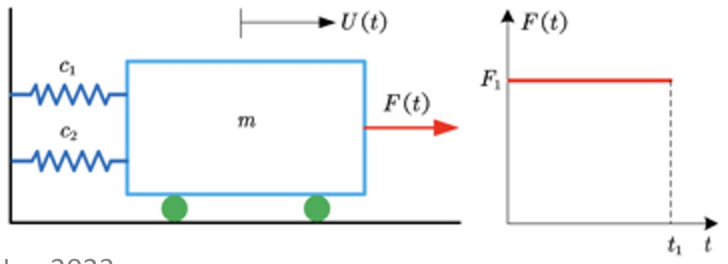
Dang, C., Wei, P., Faes, M. G., & Beer, M. (2022). Bayesian probabilistic propagation of hybrid uncertainties: Estimation of response expectation function, its variable importance and bounds. Computers & Structures, 270, 106860.

Parallel Bayesian quadrature optimization

- Propagation of mixed uncertainties based on active Bayesian integration of a Gaussian Process prior over the model's response
- Teaser:
  - nonlinear oscillator with response:

$$Z = g(c_1, c_2, m, F_1, t_1) = \left| \frac{2F_1}{c_1 + c_2} \sin\left(\frac{t_1}{2} \sqrt{\frac{c_1 + c_2}{m}} \right) \right|$$

Notation	Type	Mathematical model
c_1	Random variable	$\mathcal{N}(1, 0.1^2)$
c_2	Random variable	$\mathcal{N}(0.1, 0.01^2)$
m	Random variable	$\mathcal{N}(1, 0.1^2)$
F_1	P-box variable	$\mathcal{LN}([1\ 2], [0.1\ 0.3]^2)$
t_1	Interval variable	$[0.5\ 1.5]$

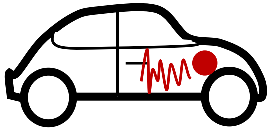


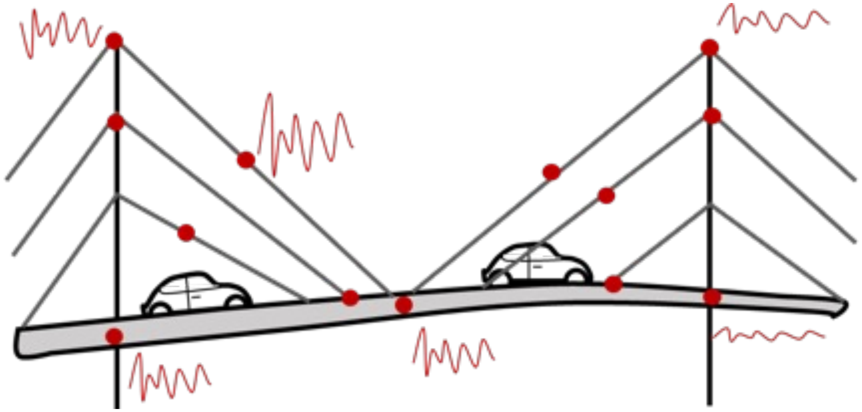
Method	$\hat{m}_l$	$\mathrm{COV}[\hat{m}_l]/\%$	$\hat{m}_u$	$\mathrm{COV}[\hat{m}_u]\ /\%$	N	$\frac{N}{c}$
DL-MCS	0.4953	0.87	2.5766	0.37	10^6	-

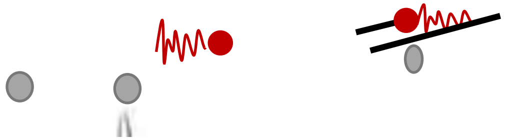
Active Bayesian optimization: teaser results

- More details and derivations in these papers:
  - C. Dang, P. Wei, M. G. R. Faes, M. A. Valdebenito, and M. Beer, 'Interval uncertainty propagation by a parallel Bayesian global optimization method', Applied Mathematical Modelling, vol. 108, pp. 220–235, Aug. 2022, doi: 10.1016/j.apm.2022.03.031.
  - C. Dang, P. Wei, M. G. R. Faes, M. A. Valdebenito, and M. Beer, 'Parallel adaptive Bayesian quadrature for rare event estimation', Reliability Engineering & System Safety, vol. 225, p. 108621, Sep. 2022, doi: 10.1016/j.ress.2022.108621.
  - C. Dang, P. Wei, M. G. R. Faes, and M. Beer, 'Bayesian probabilistic propagation of hybrid uncertainties: Estimation of response expectation function, its variable importance and bounds', Computers & Structures, vol. 270, p. 106860, Oct. 2022, doi: 10.1016/j.compstruc.2022.106860.
  - C. Dang, M. A. Valdebenito, M. G. R. Faes, P. Wei, and M. Beer, 'Structural reliability analysis: A Bayesian perspective', Structural Safety, vol. 99, p. 102259, Nov. 2022, doi: 10.1016/j.strusafe.2022.102259.

Monitoring and modelling complex systems for remaining useful life assessment under **uncertainty, nonlinearity** and **sparse noisy data**.







1. Tracking the health status

2. Making inference on future health conditions

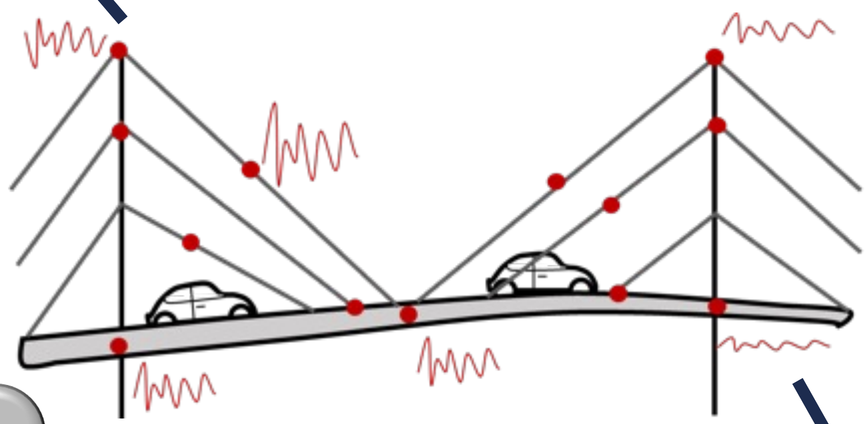
3. Assessing Safe Residual Life (SRL)

Need of IP for Safe Residual Life assessment:

- to use of unexploited information, measurements and models and integrate them!
- to quantify the uncertainty caused by the lack of knowledge

Limited knowledge on the actual structural condition (e.g. cracks, degradation)

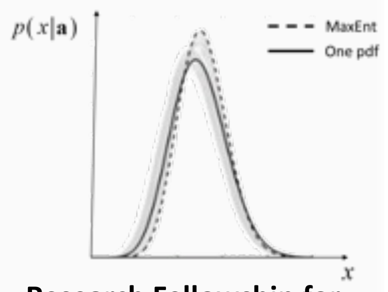
Limited measurements (e.g. settlements/accelerations at specific locations)



Uncertainty in operating and environmental conditions (time-varying)

Heterogeneous information (e.g. models of different fidelity focusing on phenomena occurring at different time-scales; inspection reports; local testing; experts' opinions; measurements of different quality/time and space resolution)

Projects on IP in Safe Residual Life assessment



Research Fellowship for Experienced Researchers

Alexander von Humboldt
Stiftung/Foundation

INSTITUTE FOR RISK AND RELIABILITY

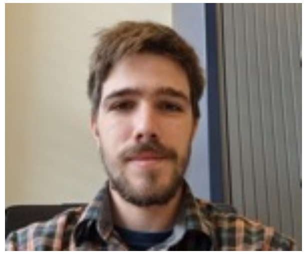
Leibniz Universität Hannover

technische universität dortmund

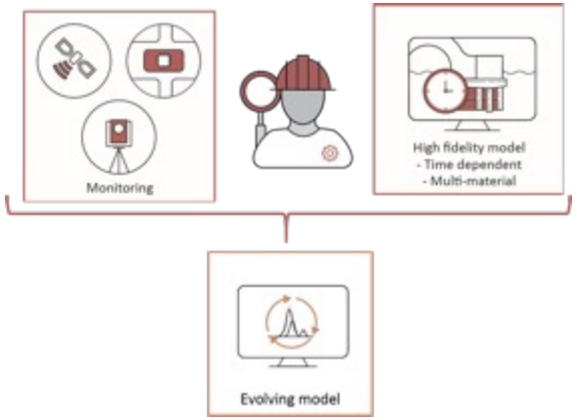
Statistical model updating strategies accounting for mixed and limited information.

NWO

LiveQuay
Live Insight in Bridges and Quay Walls



Mr Jan Koune, PhD student



Evolving probabilistic physics-informed machine learning model based on imprecise probability

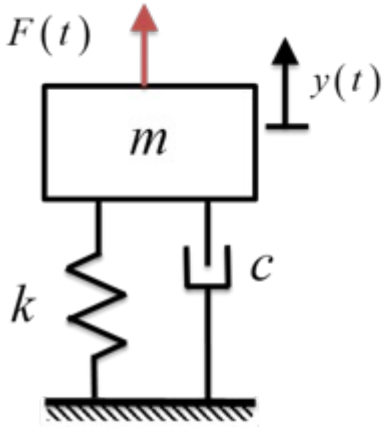
Example: Random vibration analysis of aircraft/spacecraft structural components under imprecise probability

https://www.wired.com/wp-content/uploads/images_blogs/autopia/2013/06/777assemblyline021-660x440.jpg

- Usually only the randomness in the excitation (of mechanical and acoustic nature) is of concern.
- Parameters of the structural components are assumed to be deterministic
- The random vibration analysis of aircraft/spacecraft structural components (or secondary structures) is often performed with simplified techniques, such as **Miles' Equation** (hp: dominant natural frequency with a with respect to the structural response)

Probability of exceedance of a threshold

A structural component with a dominant natural frequency with respect to the structural response



$$\sigma_y = y_{rms} = \sqrt{\frac{Q S_0(\bar{f}_n)}{32\pi^3 \bar{f}_n^{\,3}}}$$

Rms of the mass displacement

$$\omega_n = \sqrt{k/m} = 2\pi \bar{f}_n$$

$$Q = 1/(2\zeta) = (\sqrt{k\,m})/c$$

$$S_{FF}(\bar{f}) = S_0$$ broadband random loading with a constant spectral density

$$m = 0.81\ kg$$

$$Q = 10$$

$$S_0 = 100\ (\mathrm{N}/kg)^2/\mathrm{Hz}$$

Probability that the displacement $y(t)$ exceeds a given **level b** with a velocity $\dot{y}(t)$ in a small interval of time

$$P_f = \int_0^{\infty} \int_{b-\dot{y}dt}^{b} p(y,\dot{y})\mathrm{d}\dot{y}\mathrm{d}y$$

stationary Gaussian random process:

$$P_f = 1 - \exp\left[-\nu_b^+ \tau\right]$$

$$\nu_b^+ = \frac{\sigma_{\dot{y}}}{2\pi\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{b}{\sigma_y}\right)^2\right]$$

the average number of positive crossing per unit of time of a barrier

$$\sigma_{\dot{y}} = \sqrt{\frac{S_0}{8\zeta(2\pi \bar{f}_n)}}$$

$$b = 7 \times \sigma_y = 0.0018\ mm$$

$$\tau = 3\ \text{hours}$$

Imprecisely known dominant frequency!

Typical information available in Engineering

$$v_{j,\min} \le v_j = \mathrm{E}\left[f_j(x)\right] = \int f_j(x)\, p(x)\,\mathrm{d}x \le v_{j,\max}, \qquad j = 2,3,\ldots,n$$

$f_j(x) = x$ mean value

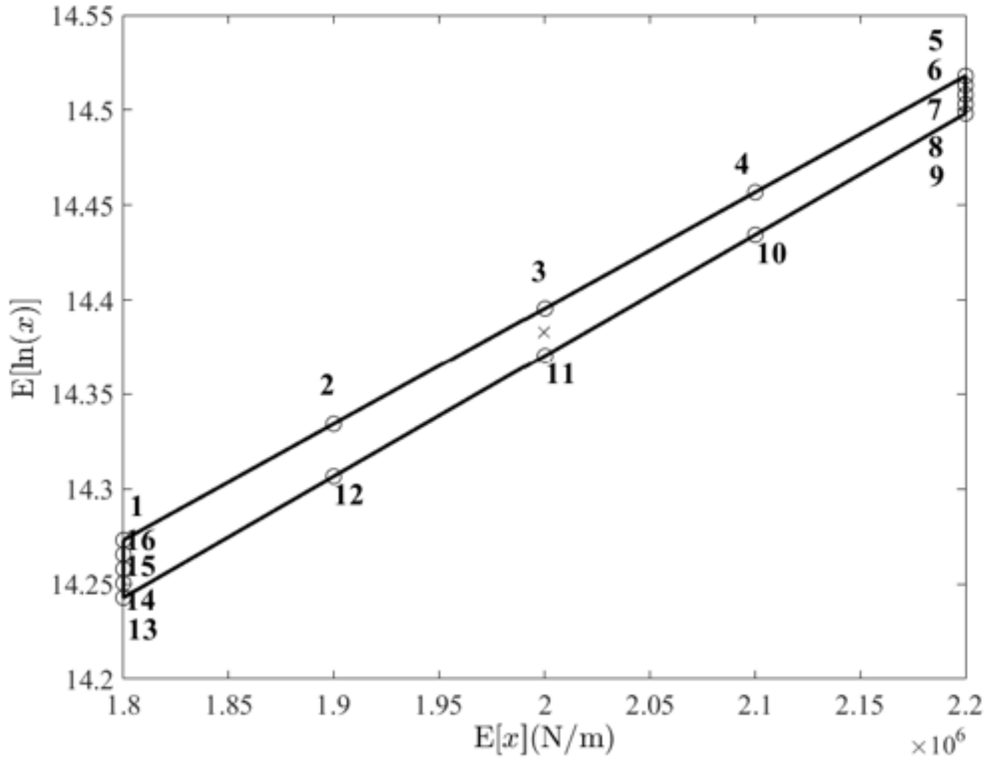
$f_j(x) = x^2$ second moment

$f_j(x) = [b,c]$ possible values that the uncertain variable may take

the constraints corresponds to the probability of finding *x* within those bounds

Example: information available on the spring stiffness

(i) the variable is positive
(ii) the vertices of a convex region of the statistical expectations are as shown



$$v_{j,\min} \leq v_j = \mathrm{E}\left[f_j(x)\right] = \int f_j(x)\,p(x)\,\mathrm{d}x \leq v_{j,\max}, \qquad j = 2,3,\ldots,n$$

Cicirello, A. and Langley, R.S. Probabilistic assessment of performance under uncertain information using a generalised maximum entropy principle, Probabilistic Engineering Mechanics, 2018. https://doi.org/10.1016/j.probengmech.2017.07.006

From equalities to the Maximum entropy distribution

$$v_j = \mathrm{E}\left[f_j(x)\right] = \int f_j(x)\, p(x)\,\mathrm{d}x \le v_{j,\max}, \qquad j = 2,3,\ldots,n$$

Maximum entropy principle:
- incorporating the current state of knowledge
- subjective pdf

$$H = -\int_{-\infty}^{+\infty} p(x)\log\left(\frac{p(x)}{t(x)}\right)\mathrm{d}x$$

$$-\int_{-\infty}^{+\infty} p(x)\log\left(\frac{p(x)}{t(x)}\right)\mathrm{d}x + \sum_{j=1}^{n}\lambda_j\left\{\int_{-\infty}^{+\infty} f(x)\,p(x)\,\mathrm{d}x - \mathrm{E}\left[f_j(x)\right]\right\}$$

$$p(x) = t(x)\exp\left[\sum_{j=1}^{n}\lambda_j f_j(x)\right].$$

$$f_1(x) = 1 \qquad \mathrm{m}_1 = \mathrm{E}\left[f_1(x)\right] = 1$$

$$\lambda_1 = \lambda_1' - 1$$

From inequalities to a Generalised Maximum entropy distribution

$$v_j = \mathrm{E}\left[f_j(x)\right] = \int f_j(x)\,p(x)\,\mathrm{d}x \leq v_{j,\max}, \qquad j = 2,3,\ldots,n$$

$$p(x|\mathbf{a} \in R) = t(x)\exp\left[\sum_{j=1}^{n} a_j f_j(x)\right]. \qquad \begin{aligned} f_1(x) &= 1 \\ t(x) &= 1 \end{aligned}$$

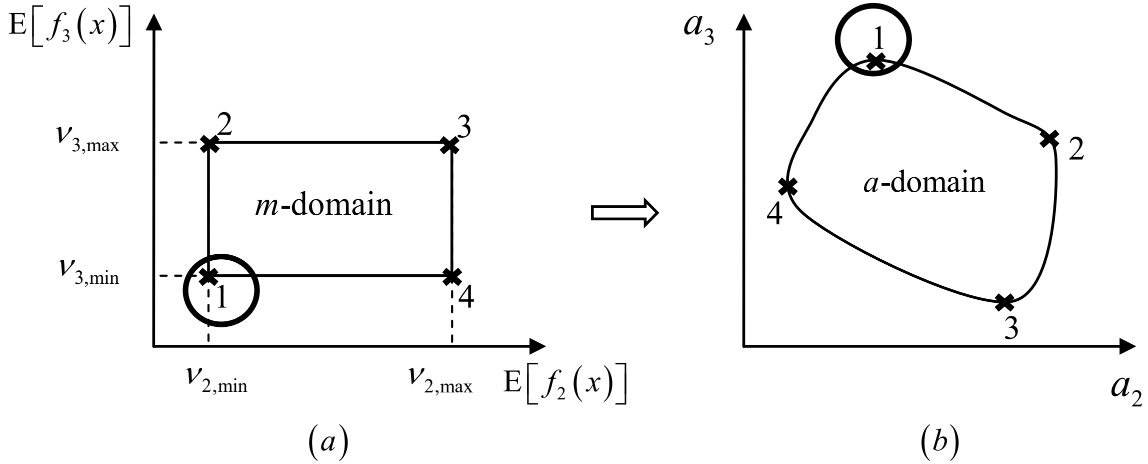
A **family of maximum entropy distributions** defined over the set of basic variables

The basic variables:

- substitute the Lagrange multipliers
- can have any possible pdf within certain bounds, including the extreme case of a delta function at any point between the bounds

Cicirello, A. and Langley, R.S. Probabilistic assessment of performance under uncertain information using a generalised maximum entropy principle, Probabilistic Engineering Mechanics, 2018. https://doi.org/10.1016/j.probengmech.2017.07.006

Mapping from the statistical bounds to the basic variables



$$p\left(x \mid \mathbf{a} \in R\right) = \exp\left[\sum_{j=1}^{n} a_j f_j\left(x\right)\right]$$

Family of pdfs

$$p\left(x \mid \mathbf{a}\right) = \exp\left[a_1 + a_2 f_2\left(x\right) + a_3 f_3\left(x\right)\right]$$

Mapping of each point

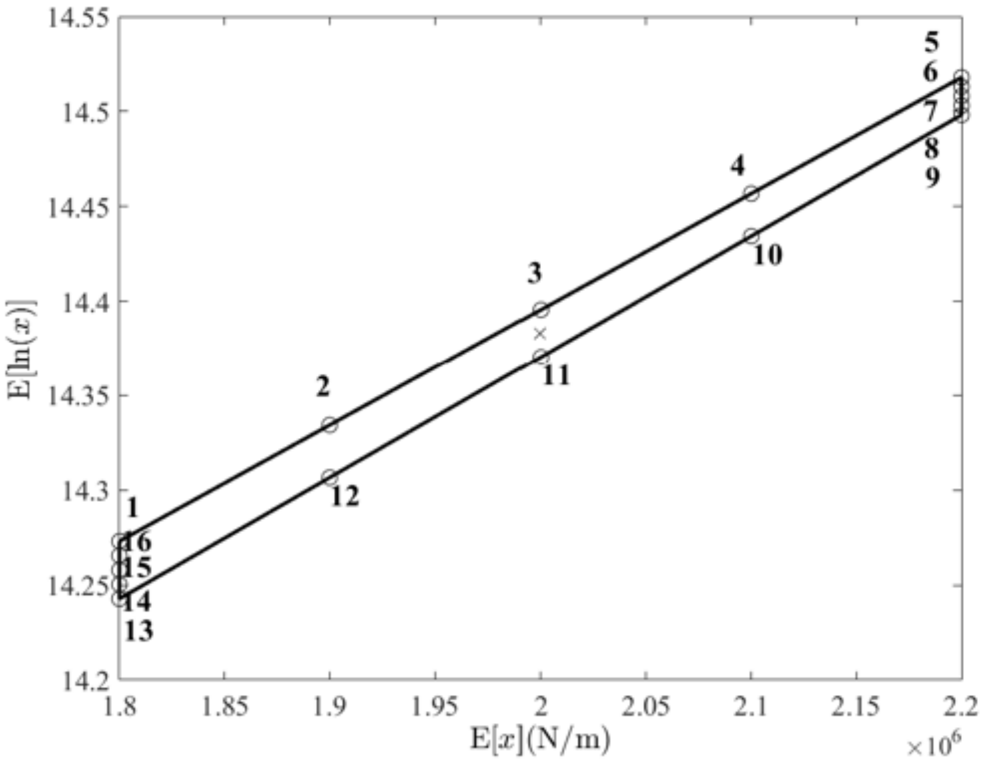
$$\begin{cases} \int f_2\left(x\right)\exp\left[a_1\left(a_{2,1}, a_{3,1}\right) + a_{2,1} f_2\left(x\right) + a_{3,1} f_3\left(x\right)\right]\mathrm{d}\,x = v_{2,\min} \\ \int f_3\left(x\right)\exp\left[a_1\left(a_{2,1}, a_{3,1}\right) + a_{2,1} f_2\left(x\right) + a_{3,1} f_3\left(x\right)\right]\mathrm{d}\,x = v_{3,\min} \end{cases}$$

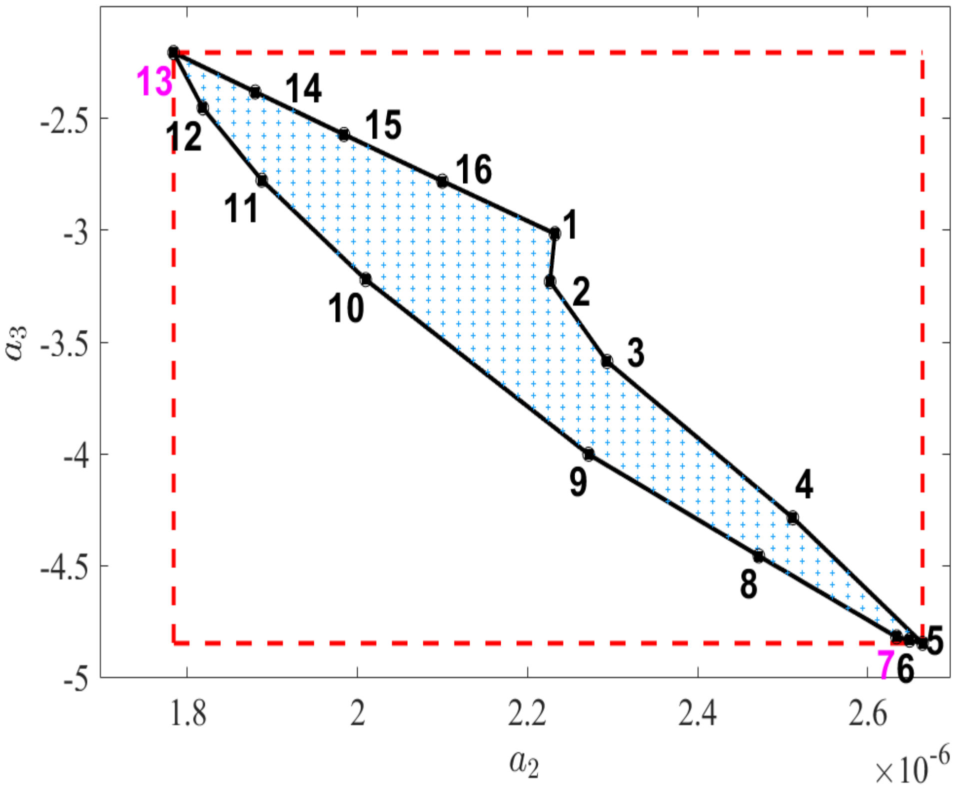
Efficient procedure was developed!
See paper!

Solve non-linear equations for each point to be mapped

Cicirello, A. and Langley, R.S. Probabilistic assessment of performance under uncertain information using a generalised maximum entropy principle, Probabilistic Engineering Mechanics, 2018. https://doi.org/10.1016/j.probengmech.2017.07.006

Example: mapping on the a-domain





$$p(x|\mathbf{a}) = \exp[a_1 - a_2 x - a_3 \ln(x)]$$

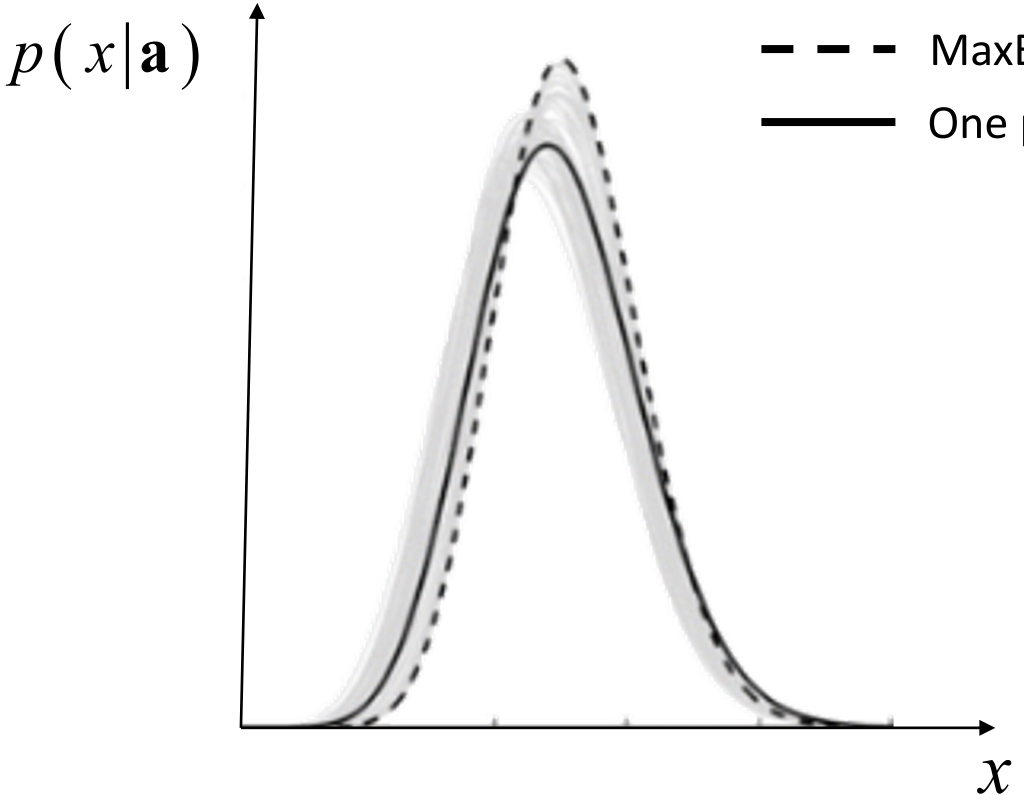
$$a_1 = -\ln\left(a_2^{(a_3-1)}\Gamma(1-a_3)\right)$$

Previous work:
- Equally spaced grid of points
- 430 points in R → 430 Pf calculations

$$0.2126 \le \overline{P}_f \le 0.4303$$

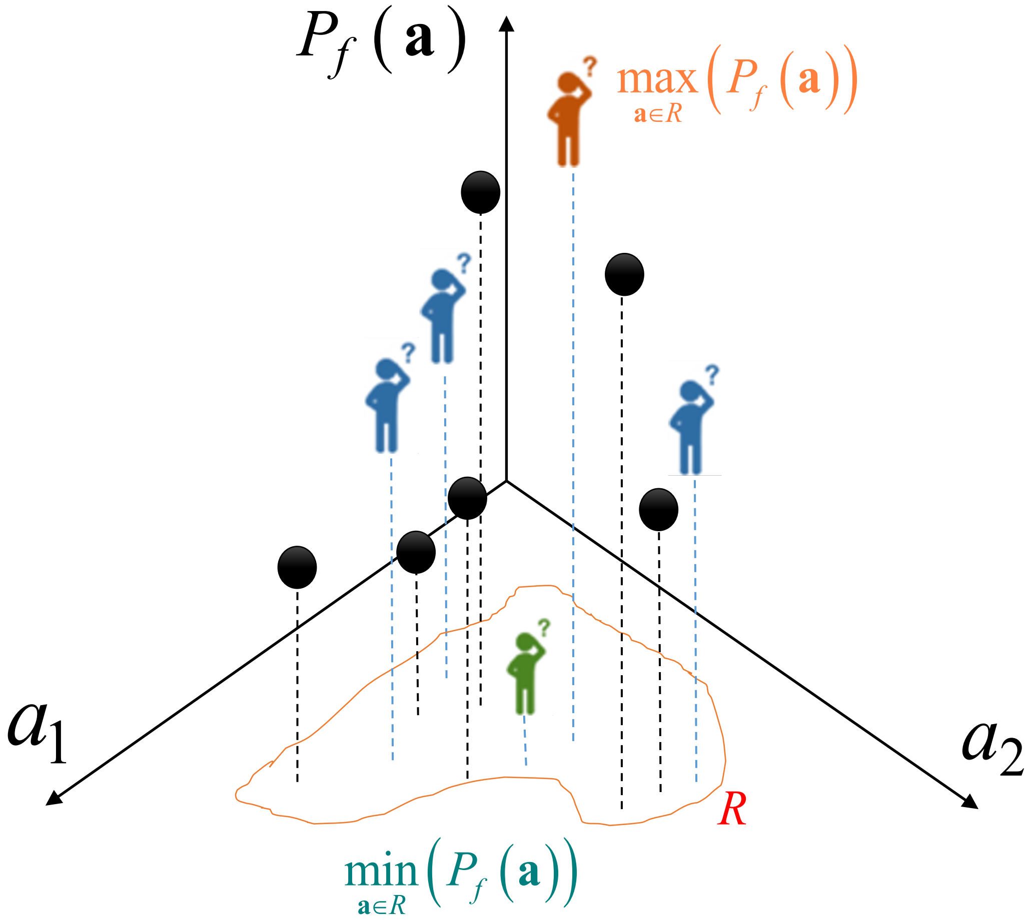
$$\overline{P}_f\Big|_{\text{point 10}} = 0.2935$$

Challenge in reliability applications: not known a-priori the pdf that will lead to max Pf!



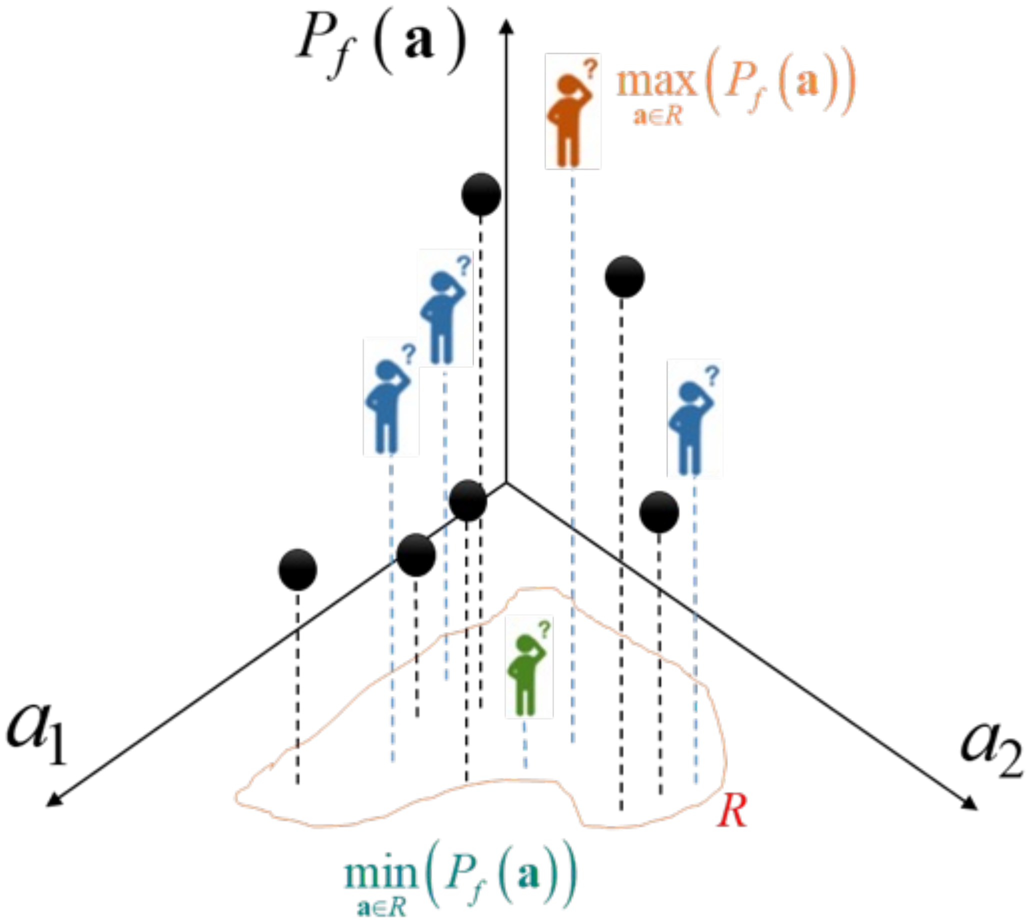
$$P_f(\mathbf{a}) = \int P_f(x|\mathbf{a})\, p(x|\mathbf{a})\,\mathrm{d}x$$

$$P_f(x|\mathbf{a}) = \int_{w_0}^{+\infty} p(w|x,\mathbf{a})\mathrm{d}x$$



$$\min_{\mathbf{a}\in R}(P_f(\mathbf{a})) \le P_f \le \max_{\mathbf{a}\in R}(P_f(\mathbf{a}))$$

Bounds on the failure probability: robust reliability analysis



$$\min_{\mathbf{a}\in R}(P_f(\mathbf{a})) \le P_f \le \max_{\mathbf{a}\in R}(P_f(\mathbf{a}))$$

The single value Pf obtained with the MaxEnt pdf can be significantly lower than the UB

The UB and LB on the chosen reliability metric can be largely different. The designer can decided to

- gain more information on the vaguely known pdf to **reduce, if possible, the basic variable domain** – would the Pf interval change a lot?
- investigate a **different design solution** which can be more robust (lowest probability interval) with respect to the uncertainty in the parameters of the distribution

Key steps: from parameterised pdf to bounds on Pf

Step 1: Evaluate few initial values of

$P_f(\mathbf{a})$

Step 2: Build a probabilistic surrogate model of

$P_f(\mathbf{a})$

Step 3: Actively select points in the a-domain to obtain bounds of Pf with very few full reliability analyses

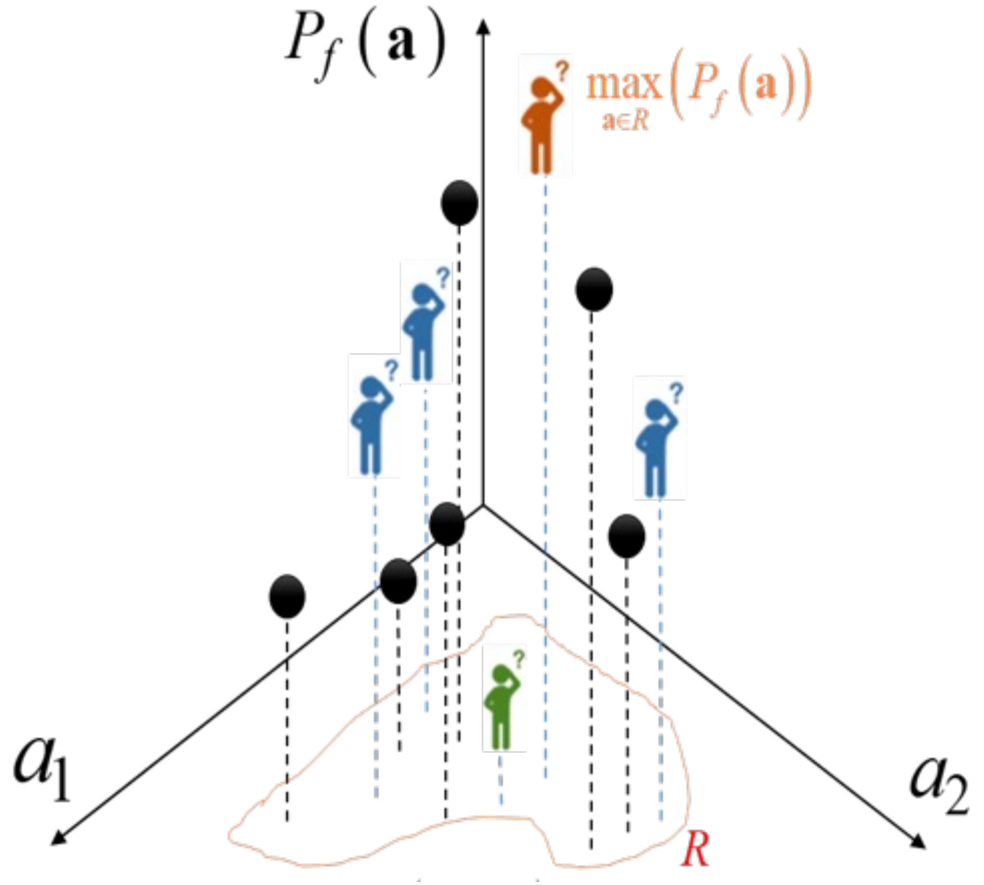
$$P_f(\mathbf{a}) = \int P_f(x|\mathbf{a})\, p(x|\mathbf{a})\,\mathrm{d}x$$

Describing the uncertainty of not having carried out the reliability analyses at each point of R

$$\min_{\mathbf{a}\in R}\left(P_f(\mathbf{a})\right) \le P_f \le \max_{\mathbf{a}\in R}\left(P_f(\mathbf{a})\right)$$

Cicirello A. Propagation of Imprecise Probability descriptions via machine learning based optimization for robust reliability analysis. In: International Symposium on Reliability Engineering and Risk Management 2022, Hannover, Germany, 2022.

Step 2: probabilistic description of $P_f(\mathbf{a})$



- **Discrete function at each aj point in R**

$$\mathbf{P_f} = \left[P_f(\mathbf{a}_1), P_f(\mathbf{a}_2), \ldots, P_f(\mathbf{a}_N)\right]$$

- **Additional uncertainty caused by not having evaluated the model**

- Model **each reliability result** that has not been computed yet as a Gaussian random variable

- **The collection of random variables is a Gaussian Process!**

$$\mathbf{P_f} = \left[\mathbf{P_{f0}}, \mathbf{P_{f*}}\right] \quad \begin{cases} D = \left\{\left(\mathbf{a}_{0j}, P_f(\mathbf{a}_{0j})\right), j = 1 : N_t\right\} \\ P_{f*k} = P_f(\mathbf{a}_{*k}) \end{cases}$$

Noise-free observations: reliability analyses carried out

Function realizations we want to predict

Step 2: Predictive mean and variance

Noise-free observations

$$\begin{pmatrix} \mathbf{P}_{\mathbf{f}0} \\ \mathbf{P}_{\mathbf{f}*} \end{pmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\mu}_0 \\ \boldsymbol{\mu}_* \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{00} & \boldsymbol{\Sigma}_{0*} \\ \boldsymbol{\Sigma}_{*0} & \boldsymbol{\Sigma}_{**} \end{bmatrix} \right)$$

Function realizations we want to predict

$$\boldsymbol{\mu}_0 = \left[m_0\left(\mathbf{a}_{01}\right), \ldots, m_0\left(\mathbf{a}_{0N_0}\right) \right] \qquad D = \left(\mathbf{A}_0, \mathbf{P}_{\mathbf{f}0}\right)$$

$$\boldsymbol{\mu}_* = \left[m_*\left(\mathbf{a}_{*1}\right), \ldots, m_*\left(\mathbf{a}_{*n_*}\right) \right] \qquad \left(\mathbf{A}_*, \mathbf{P}_{\mathbf{f}*}\right)$$

$$\left(\boldsymbol{\Sigma}_{0*}\right)_{i,j} = \mathrm{Cov}\left[P_f\left(\mathbf{a}_{0i}\right), P_f\left(\mathbf{a}_{*j}\right) \right] = \kappa\left(\mathbf{a}_{0i}, \mathbf{a}_{*j}\right)$$

Kernel function: encodes smoothness of the response function. Return similarity between two points

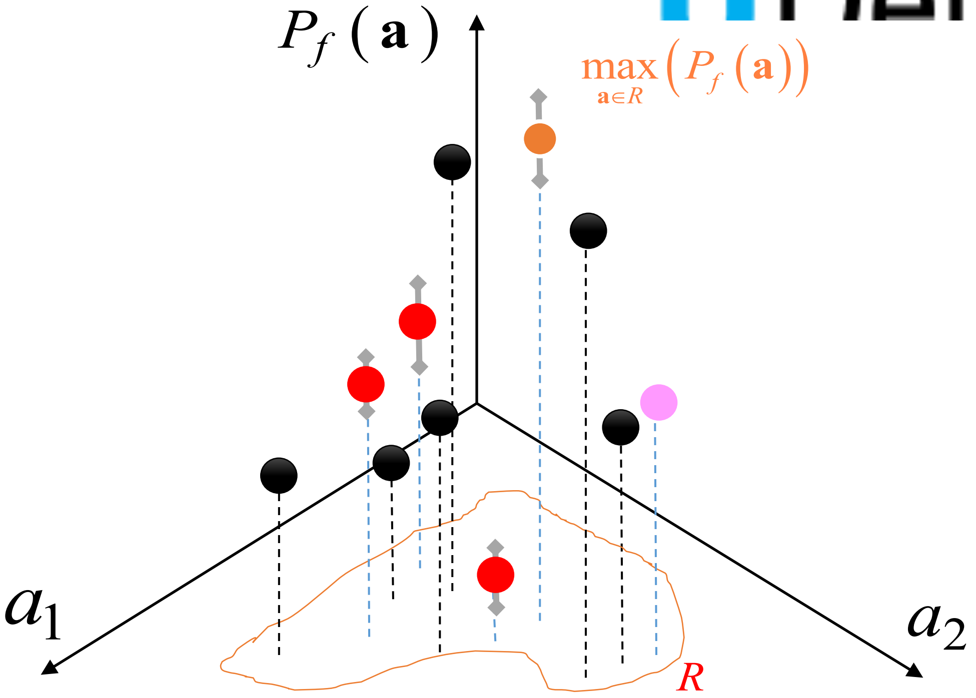
By conditioning $$p\left(\mathbf{P}_{\mathbf{f}*} \middle| \mathbf{A}_*, \mathbf{A}_0, \mathbf{P}_{\mathbf{f}0}\right) \sim N\left(\boldsymbol{\mu}_* + \boldsymbol{\Sigma}_{*0}\boldsymbol{\Sigma}_{00}^{-1}\left(\mathbf{P}_{\mathbf{f}0} - \boldsymbol{\mu}_0\right), \quad \boldsymbol{\Sigma}_{**} - \boldsymbol{\Sigma}_{*0}\boldsymbol{\Sigma}_{00}^{-1}\boldsymbol{\Sigma}_{0*} \right)$$

Only the **predictive marginal mean and variance** are needed to quantify the predictive uncertainty at each test point

$$m\left(\mathbf{a}_j\right) = m\left(\mathbf{a}_{*j}\right) + \boldsymbol{\Sigma}_{j0}\boldsymbol{\Sigma}_{00}^{-1}\left(\mathbf{P}_{\mathbf{f}0} - \boldsymbol{\mu}_0\right) \qquad \sigma^2\left(\mathbf{a}_j\right) = \kappa\left(\mathbf{a}_{*j}, \mathbf{a}_{*j}\right) - \boldsymbol{\Sigma}_{j0}\boldsymbol{\Sigma}_{00}^{-1}\boldsymbol{\Sigma}_{0j}$$

Mean value at each test point **Variance value at each test point**

Active learning: from training dataset to UB



Initial Training Dataset: few full reliability analyses carried out for N_t points in R ●

UB response estimation

GP regression Model (using all available data)

Stopping criteria

Selection of one point ● in R for improving UB

Enhanced Training Dataset UB

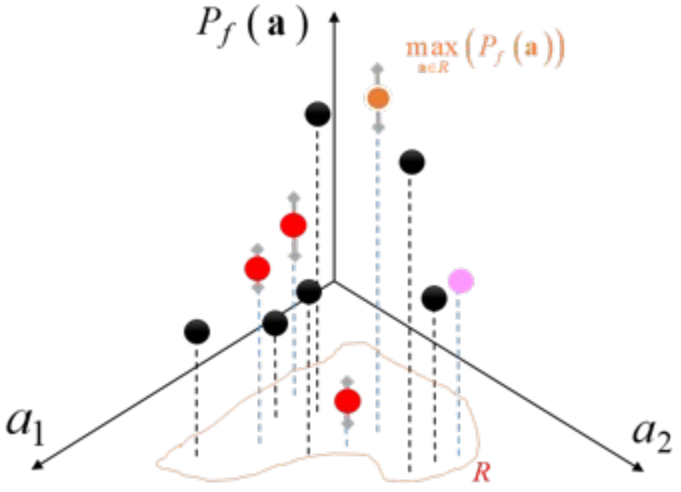
iteration for UB

Cicirello A. Propagation of Imprecise Probability descriptions via machine learning based optimization for robust reliability analysis. In: International Symposium on Reliability Engineering and Risk Management 2022, Hannover, Germany, 2022.

Active learning: next point selection and stopping

Selection of one point in R for improving UB

$$P_{f0,\max} = \max\left[\mathbf{P}_{\mathbf{f}0}\right]$$



Improvement

$$I^{(+)}(\mathbf{a}) = \begin{cases} P_f(\mathbf{a}) - P_{f0,\max} & \text{if } P_f(\mathbf{a}) > P_{f0,\max} \\ 0 & \end{cases}$$

Probability of Improvement

$$p\left(I^{(+)}(\mathbf{a})\right) \sim N\left(m(\mathbf{a}) - P_{f0,\max}, \sigma^2(\mathbf{a})\right)$$

Expected Improvement

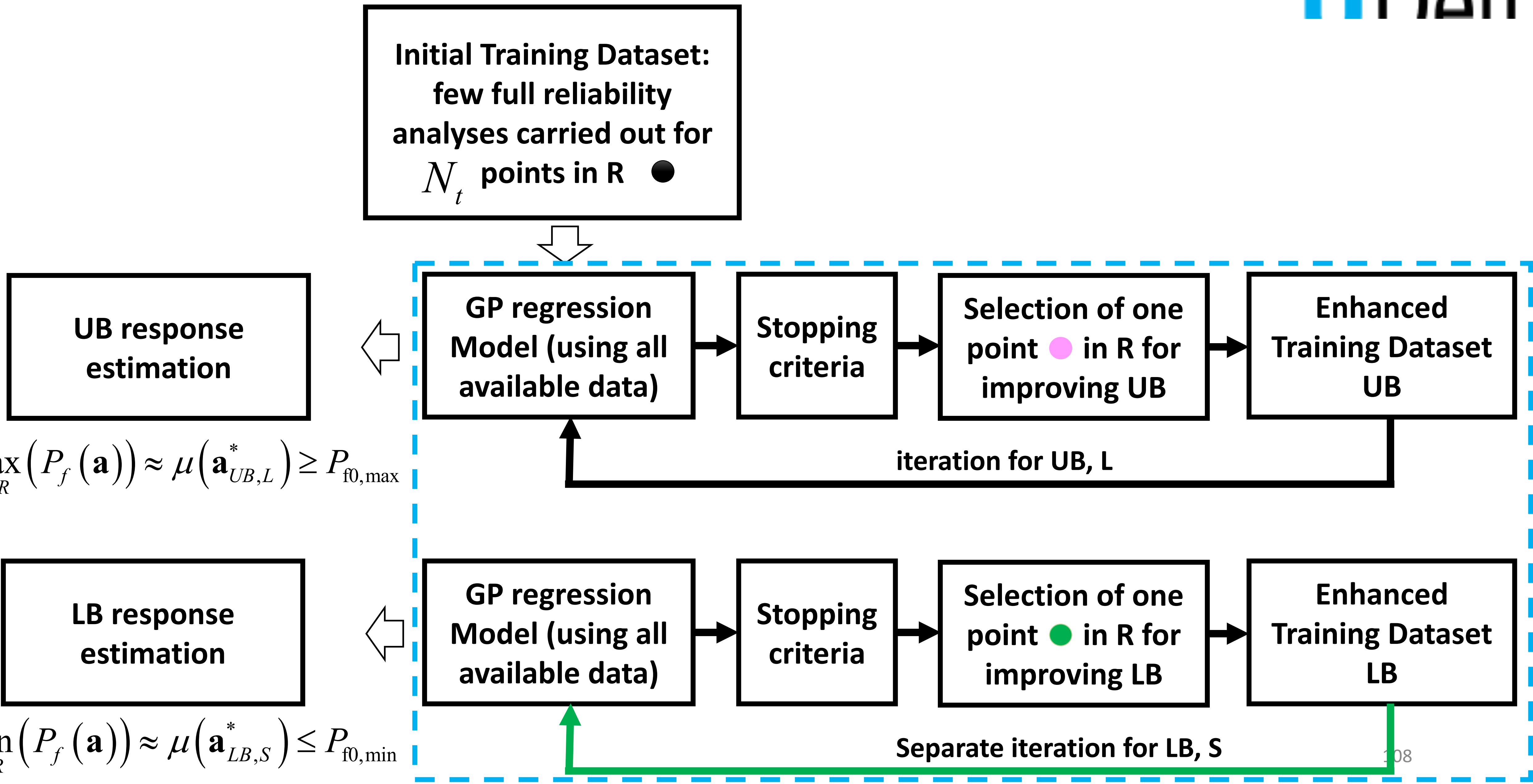
$$EI^{(+)}(\mathbf{a}) = \int_0^{\infty} I^{(+)}\, p\left(I^{(+)}(\mathbf{a})\right) dI^{(+)}.$$

$$\hat{\mathbf{a}}^*_{\max} = \arg\max_{\mathbf{a}\in R}\left[EI^{(+)}(\mathbf{a})\right]$$

Stopping criteria

- max number of full reliability analyses has been reached
- criterion satisfied on the Acquisition Function $(\Delta < 0.0001)$

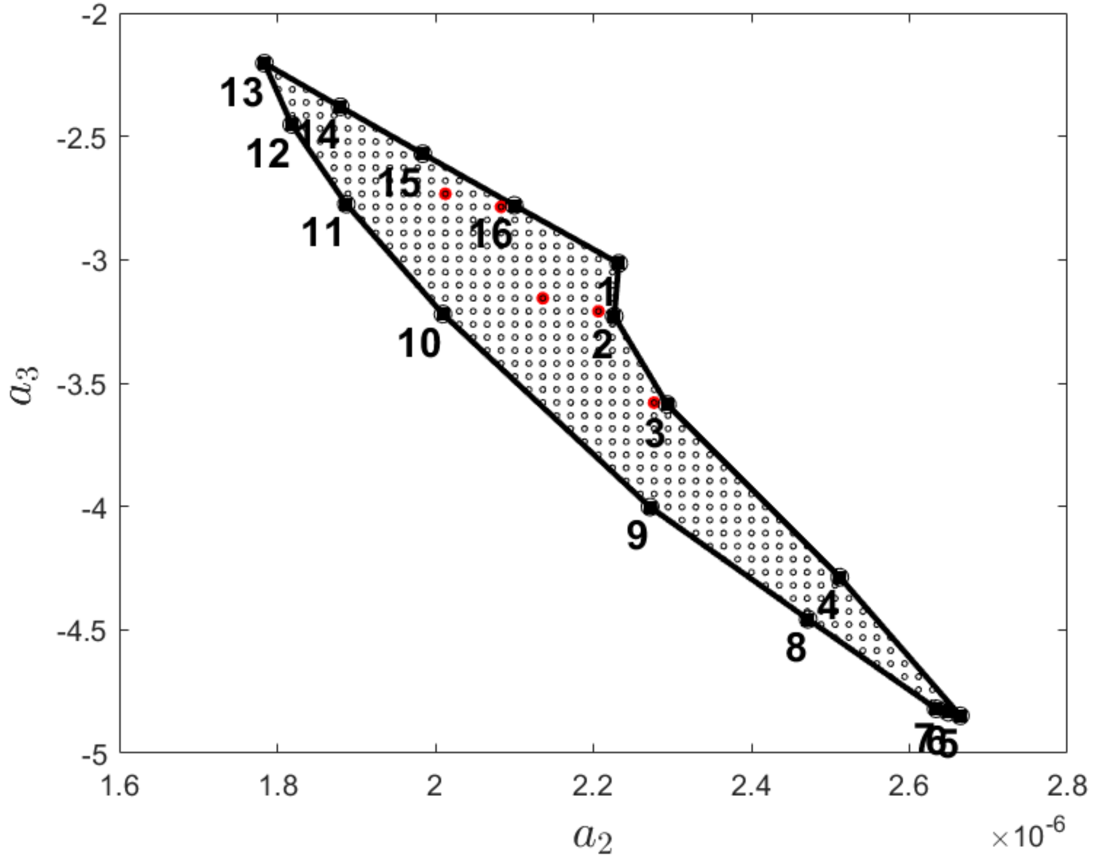
UB and LB estimation: from the GP mean function

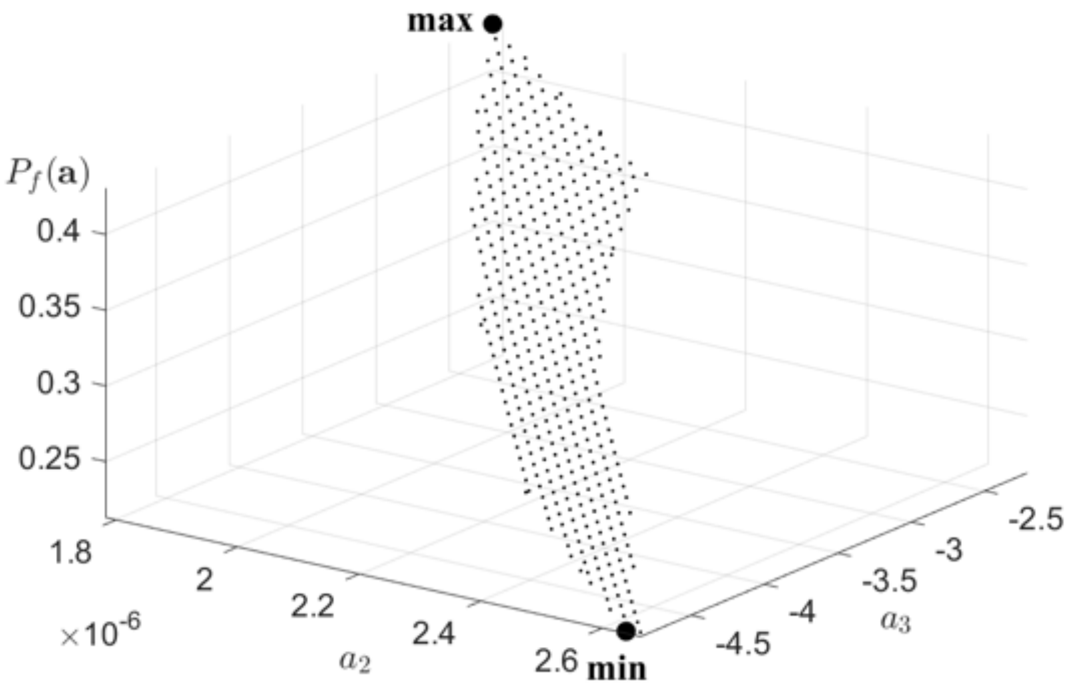


$$\max_{\mathbf{a}\in R}\left(P_f\left(\mathbf{a}\right)\right)\approx\mu\left(\mathbf{a}^*_{UB,L}\right)\geq P_{\mathrm{f0,max}}$$

$$\min_{\mathbf{a}\in R}\left(P_f\left(\mathbf{a}\right)\right)\approx\mu\left(\mathbf{a}^*_{LB,S}\right)\leq P_{\mathrm{f0,min}}$$

Results of the proposed approach





One realization of the 5 randomly selected initial points

Squared exponential kernel

Stop criteria: - max 5 simulation per bound

- $\Delta < 0.0001$

Table 2. Lower Bound and Upper Bound results.

Bound	Benchmark	Initial GP	Proposed
Lower	0.21263	0.30953	0.21263 (Iter. 5)
Error	-	45.57 %	0 %
Upper	0.43025	0.40376	0.40376 (Iter. 1)
Error	-	6.16 %	6.16 %

Total of 15 at most for other starting points!

Conclusions

From statistical bounds to a Generalised Maximum Entropy distribution

A **non-intrusive approach** for propagating imprecise probability (as a parameterised pdf) to yield bounds of Pf:

- Drastically reducing the number of full reliability analyse to be carried out
- Embeds uncertainty in the response bounds estimates arising from having run few simulations
- For the case investigated: **bounds obtained in 11 full reliability analyses, rather than 430**!

Interpretable

No modifications of the expensive-to-evaluate physics-based model and reliability model

Probabilistic model (GP) embedding both user knowledge (kernel) and noise-free observations (physics-based model + reliability model)

Selection of points in the basic variable domain justified in terms of the trade-off between exploration and exploitation

Explainable

The predicted bounds wrt to current observations:

- Overpredict the UB
- Underpredict the LB

The bounds on the UB and LB embed explicitly uncertainty of not having evaluated all the possible simulations → will be used in the future!

Why is IP not (yet!) widely used in Engineering?

What is the biggest challenge in IP for Engineering?

Intervals analysis in engineering

- Used when upper and lower bounds are known reliably
- Sometimes absolutely sure, sometimes just a judgment

- Intervals are easy to understand
- Calculations with them are easy
- They work with even the crappiest data
- Often sufficient for a decision
- Quicker than Monte Carlo
- The computations are guaranteed to enclose the true results (so long as the inputs do)
- You can still be wrong, but the *method* won't be the reason if you are

Summary

Imprecise probability allows to deal with lack of knowledge and imprecision

Rigorous propagation of uncertainty exist (intrusive and non-intrusive) and freely available libraries

Embedded feature in the digital twin and UQ a by-product