Confidence in Beliefs and Rational Policy Making

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Climate Policy & Uncertainty

Climate Policy (e.g. Social Cost of Carbon)

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$$\Uparrow$$

Equilibrium Climate Sensitivity (ECS)

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Equilibrium Climate Sensitivity (ECS)

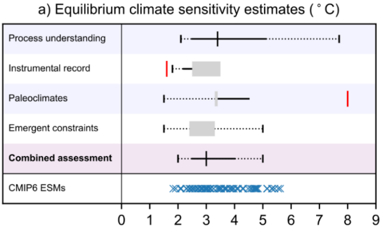


Figure: (Masson-Delmotte et al., 2021, fig TS.16b).

- Best estimate: $3^{\circ}C$
- Likely in range 2.5 to $4^{\circ}C$ (high confidence)
- Virtually certain above $1.5^{\circ}C$ (high confidence)
- Very likely below $5^{\circ}C$ (medium confidence)

(Masson-Delmotte et al., 2021)

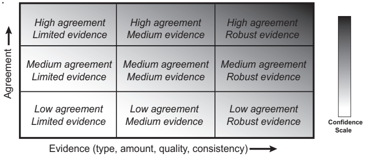
IPCC uncertainty language

ECS is likely in range 2.5 *to* 4°*C (high confidence)*

CONFIDENCE 'in the validity of a finding, based on the type, amount, quality, and consistency of evidence . . . and the degree of agreement.'

PROBABILITY 'based on statistical analysis of observations or model results, or expert judgment' ('Likelihood')

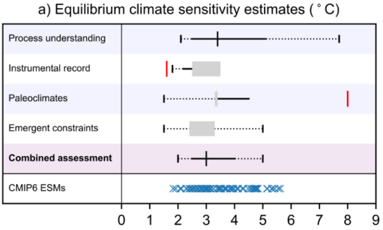
Table 1. Likelihood Scale	
Term*	**Likelihood of the Outcome**
Virtually certain	99-100% probability
Very likely	90-100% probability
Likely	66-100% probability
About as likely as not	33 to 66% probability
Unlikely	0-33% probability
Very unlikely	0-10% probability
Exceptionally unlikely	0-1% probability



Mastrandrea et al. (2010); Masson-Delmotte et al. (2021)

Climate Policy & Uncertainty

How to decide when there is this much uncertainty?



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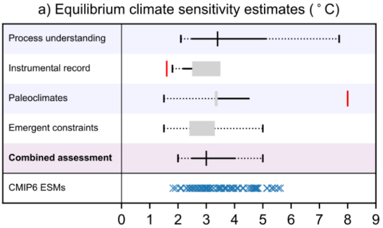
Climate Policy & Uncertainty

Aim Overview of issues & a proposal concerning:

How to decide when there is this much uncertainty?



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Plan

What is it? Three tenets:

Belief Beliefs (and hence uncertainty) can be represented by **probabilities.**

Decision Decision Makers maximise **expected utility.**
Subjective Expected Utility (SEU)

Belief Formation Update by **conditionalisation.**

Expected utility

$$\mathbb{E}_p(u(f)) = \sum_{s \in S} u(f(s)).p(s)$$

- u: utility function, representing desires or tastes
- p: probability measure, representing beliefs

Bayesianism

A Quick Introduction

What is it? Three tenets:

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- **Belief Formation** Update by **conditionalisation.**

Saying something:

- Descriptive
- Normative
  - guide the rationalisation, evaluation and deliberation of (tough) decisions and uncertainty

Why? Purportedly:

1. has acceptable choice-theoretical consequences
2. is conceptually clear about the roles of different mental attitudes

Why? Purportedly:

1. has acceptable choice-theoretical consequences
2. is conceptually clear about the roles of different mental attitudes

- Dutch Book Arguments, Representation Theorems

i.e. results of the following form:

Properties of preferences / choice $\Leftrightarrow$ $\exists\ p, u$ s.t. choice maximises $\mathbb{E}_p(u(f))$

And the p, u are appropriately unique.

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Bayesianism

A Quick Introduction

Why? Purportedly:

1. has acceptable choice-theoretical consequences
2. is conceptually clear about the roles of different mental attitudes

And also Provides framework for:

- belief update
- uncertainty reporting
- belief aggregation
- belief elicitation
- decision analysis

Bayesianism and Climate Uncertainty

The lack of firm probabilities is not a reason to give up expected value theory. You might despair and adopt some other way of coping with uncertainty . . . . That would be a mistake. Stick with expected value theory . . . and do your best with probabilities and values. (Broome, 2012)

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→ Just pick a (precise) probability and use it

The Challenge(s)

Unknown urn	Known urn
100 balls	100 balls
Each red or black	50 red, 50 black

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What do you **believe** about the next ball drawn?

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indifferent.

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Ellsberg:
Known urn

The Challenge(s)

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Bayesianism denies any role to (something like)
weight of evidence
in choice

Severe Uncertainty

Project

Belief Representation & Decision

Aim

To formulate and defend:

- Normatively valid models to give guidance for rational decision making under uncertainty.

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Develop the framework to account for:

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Belief Representation & Decision

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To formulate and defend:

- Normatively valid models to give guidance for rational decision making under uncertainty.

Desiderata

1. it has acceptable choice-theoretical consequences
2. it is conceptually clear about the roles of different mental attitudes

And of course:

3. it can fruitfully deal with real-life “severe” uncertainty situations such as those above.

Plan

Proposal in a nutshell

The concept . . .

> *to express the proper state of belief, not* one *number but* two *are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based. (Peirce, 1878, p179)*

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Beyond the degree to which one endorses a particular proposition

. . . there is the degree to which one is confident in this endorsement.

If the former is one's beliefs, the latter is one's

confidence in one's beliefs

Modelling confidence

Starting point Represent beliefs by

- A single probability measure (Bayesianism)

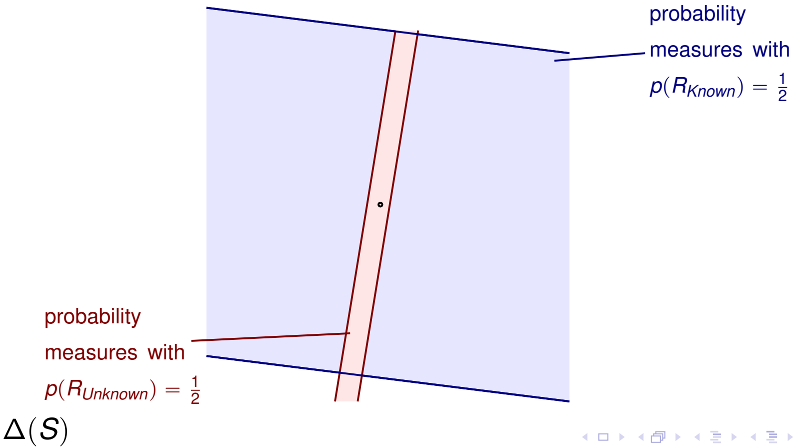
$\Delta(S)$

Modelling confidence

Starting point Represent beliefs by

- A single probability measure (Bayesianism)

≡ A complete, consistent set of probability judgements (e.g. 'the probability of A is no smaller than x')

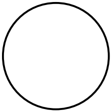


Modelling confidence

First attempt Represent beliefs by

- A set of probability measures (multiple priors, credal sets)

≡ A consistent set of probability judgements

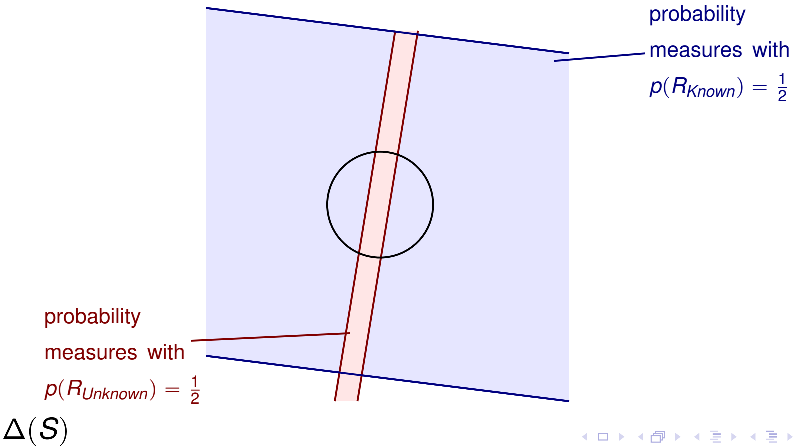


$\Delta(S)$

Modelling confidence

Interpretation

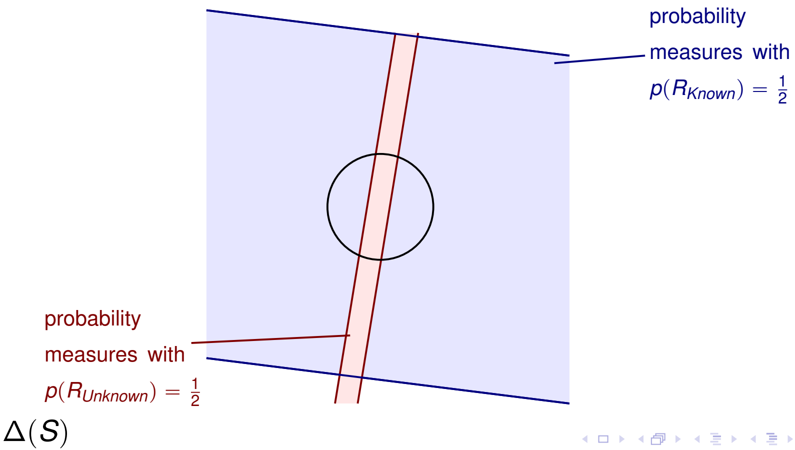
- confident that the probability of R_{Known} is 0.5
- unsure about whether the probability of $R_{Unknown}$ is 0.5



Modelling confidence

Problem (credal sets)

- confidence is represented as “binary”.
- but it comes in degrees.



Modelling confidence

Idea Represent beliefs by

- A consistent set of probability judgements for each confidence level

$\Delta(S)$

Modelling confidence

Idea Represent beliefs by

- A nested family of sets of measures

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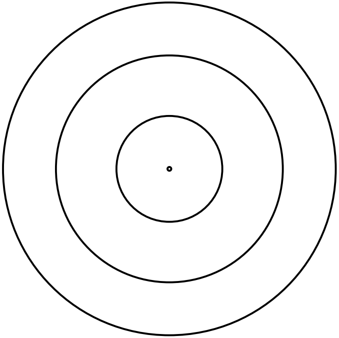

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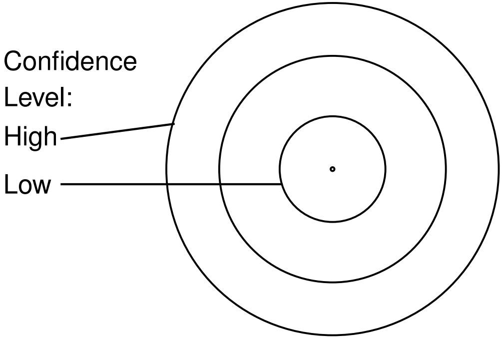
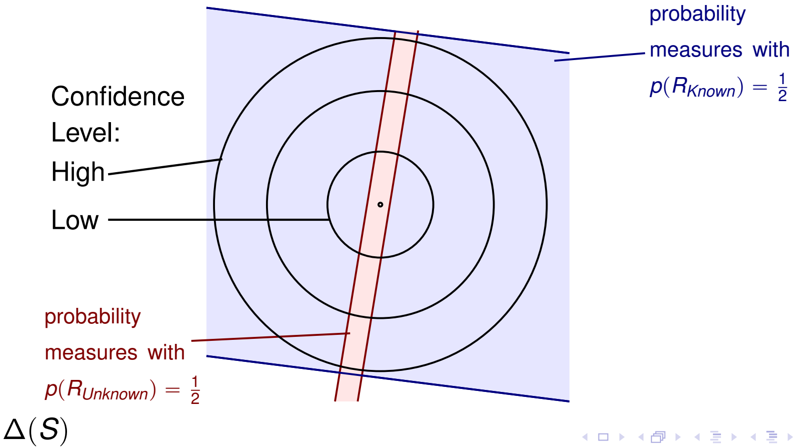


$\Delta(S)$

Modelling confidence

Interpretation

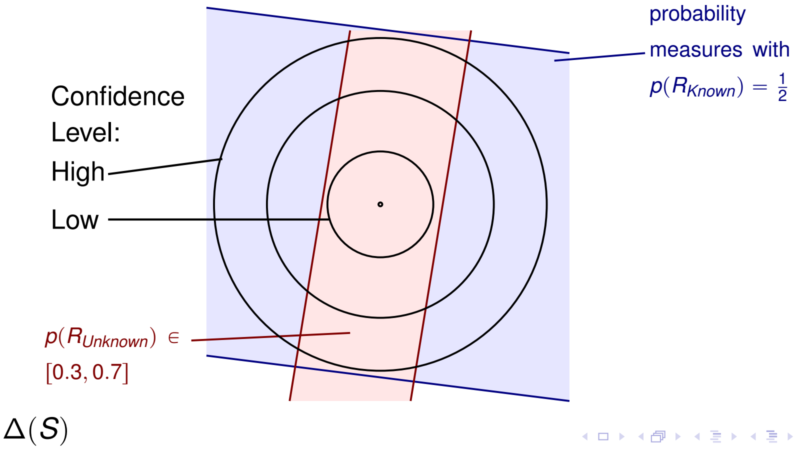
- **more confident** in $p(R_{Known}) = 0.5$ than $p(R_{Unknown}) = 0.5$



Modelling confidence

Interpretation

- **more confident** in $p(R_{Unknown}) \in [0.3, 0.7]$ than $p(R_{Unknown}) = 0.5$



Modelling confidence

Definition

A **confidence ranking** Ξ is a nested family of closed subsets of $\Delta(S)$.

Centered: it contains a singleton set (Bayesian with confidence).

Convex: every $C \in \Xi$ is convex.

Continuous: for every $C \in \Xi$, $C = \overline{\bigcup_{C' \subsetneq C} C'} = \bigcap_{C' \supsetneq C} C'$.

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Lifting: High confidence in judgement $\Rightarrow$ held at lower confidence levels; not vice versa.

IPs: At each confidence level, standard logic of credal sets.

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IPs: At each confidence level, standard logic of credal sets.

- Confidence-imprecision tradeoff:
  - Confidence level $\uparrow$, imprecision $\uparrow$.

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Equivalent representations

O: (ordered) set of confidence levels.

Conf. ranking $c : O \to 2^{\Delta}$

Implausibility fn $\iota : \Delta \to O$

Conf. in judg. $conf : 2^{\Delta} \to O$

$$\iota(p) = \min \{o : p \in c(o)\}$$

$$conf(\mathcal{P}) = \max \{o : c(o) \subseteq \mathcal{P}\}$$

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Also

- $O \to$ sets of desirable gambles, coherent lower previsions, belief functions etc.

Confidence and Decision

There appear to be many significant decisions where confidence in beliefs do, and should, play a role.

The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself. (Knight, 1921, p226-227)

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But what role?

Proposal in a nutshell

. . . and the intuition

- would we like decisions about climate change policy to be taken on the basis of “best hunch” estimates?

Proposal in a nutshell

. . . and the intuition

- would we like decisions about climate change policy to be taken on the basis of “best hunch” estimates?
- and what about wagers between us?

Proposal in a nutshell

. . . and the intuition

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

The role of confidence in choice

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Decision

f

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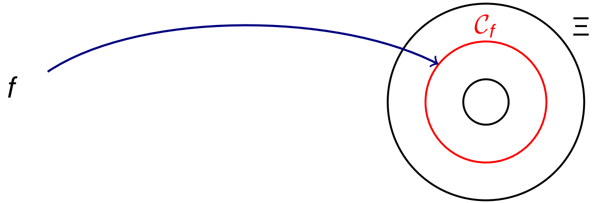
Confidence level



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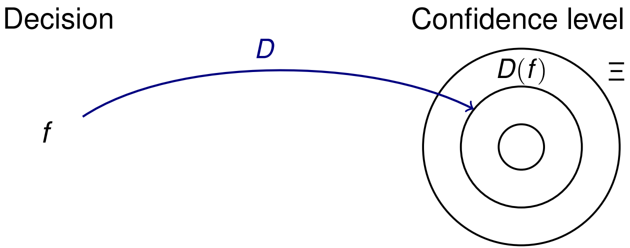

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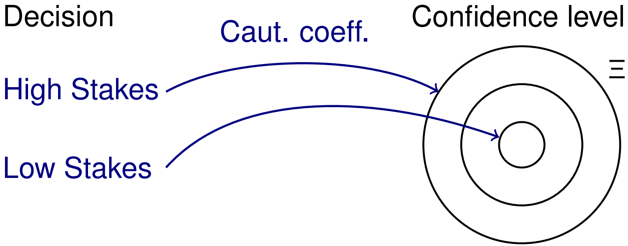

A **cautiousness coefficient** for a confidence ranking Ξ is a surjective function $D : \mathcal{A} \to \Xi$

- the higher the stakes, the larger $D(f)$

Decision

The Confidence Framework

Ingredients:

- utility function u
- confidence ranking Ξ
- cautiousness coefficient D
- decision rule I

General form:

preferences concerning f are a function of

$u(f(s))$ and $D(f)$

according to decision rule I

Confidence and choice

Examples

Decision rules using a (closed) set of probabilities:

- unanimity rule

  $f \preceq g \quad \text{iff} \quad \mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g)) \quad \text{for all } p \in \max\{D(f), D(g)\}$

- maxmin expected utility

  $f \preceq g \quad \text{iff} \quad \min_{p \in D(f)} \mathbb{E}_p(u(f)) \leqslant \min_{p \in D(g)} \mathbb{E}_p(u(g))$

- Hurwicz or α-maxmin rule

  $\alpha \min_{p \in D(f)} \mathbb{E}_p(u(f)) + (1-\alpha) \max_{p \in D(g)} \mathbb{E}_p(u(f))$

- etc.

On stakes

Confidence and chance

Example: Careful preferences

- “Maxmin EU” decision rule

Confidence and chance

Example: Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Confidence and chance

Example: Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

NB:

- $D(f)$ is a belief function $\Rightarrow$ convex capacity $\overset{\textit{monotonicity}}{\Longrightarrow}$ Choquet EU $\equiv$ maxmin EU (over core)

Confidence and chance

Example: Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Under such a rule:

- for higher stakes, one is effectively only relying on beliefs in which one has sufficient confidence.
- behaviour is as "pessimistic" as one's confidence: the more confident in appropriate beliefs or the lower the stakes, the less pessimistic.

Confidence and chance

Example: Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Conclusion This yields the following advice:

Choose boldly if one has sufficient confidence; choose cautiously if not.

Confidence and chance

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Conclusion This yields the following advice:

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Comparison Few "non-EU" rules correspond so closely to plausible maxims of this sort.

Comparison This rule is not as extreme as maxmin EU.

Ellsberg Can accommodate Ellsberg behaviour in the same way as the "standard maxmin EU rule".

Incomplete Preferences

Plan

Desiderata

1. it has acceptable choice-theoretical consequences
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And of course:

✓ it can fruitfully deal with "severe uncertainty" situations such as those above.

Desiderata

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▸ Dutch Books ▸ Representation Theorems
▸ Neither in detail

Dynamic choice-theoretic arguments: see Hill (2020) for a rebuttal.

A typical framework

S	non-empty finite set of states
$\Delta(S)$	set of probability measures on S
C	set of consequences
$\mathcal{A}$	set of acts (functions $S \to C$)
$\preceq$	preference relation on $\mathcal{A}$

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Notation:

- $f_\alpha g$: shorthand for $\alpha f + (1 - \alpha) g$.

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Special case (Anscombe-Aumann framework)

$C =$ set of lotteries (probability distributions with finite support) over a set X of prizes.

Careful preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{\alpha \in [0, 1] | \, f_\alpha h \preceq g\}$ and $\{\alpha \in [0, 1] | \, f_\alpha h \succeq g\}$ are closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

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Careful preferences

Axioms

Confidence-based careful preference model:

For all $f, g, h \in \mathcal{A}$, $c, d \in C$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

S-Independence (i) if $f \succeq c$, then $f_\alpha d \succeq c_\alpha d$ whenever the stakes are lower.
(ii) if $f \preceq c$, then $f_\alpha d \preceq c_\alpha d$ whenever the stakes are higher.

Continuity $\{\alpha \in [0,1] | \, f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | \, f_\alpha h \succeq g\}$ are closed.

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Careful preferences

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Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

S-Independence (i) if $f \succeq c$, then $f_\alpha d \succeq c_\alpha d$ whenever the stakes are lower.

(ii) if $f \preceq c$, then $f_\alpha d \preceq c_\alpha d$ whenever the stakes are higher.

Continuity $\{\alpha \in [0, 1] | f_\alpha h \preceq g\}$ and $\{\alpha \in [0, 1] | f_\alpha h \succeq g\}$ are closed.

Monotonicity applies to acts of the same stakes

Uncertainty Aversion applies to acts of the same stakes

Careful preferences

Representation theorem

Theorem

$\preceq$ satisfies axioms above

$\Leftrightarrow$ *there exist $u : X \to \Re$, Ξ and $D : \mathcal{A} \to \Xi$ such that, for all $f, g \in \mathcal{A}$, $f \preceq g$ iff*

$$\min_{p \in D(f)} \mathbb{E}_p u(f(s)) \leqslant \min_{p \in D(g)} \mathbb{E}_p u(f(s))$$

Furthermore u is unique up to positive affine transformation, and Ξ and D are unique.

▸ Enough

Incomplete preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

- Non triviality and reflexivity: $\preceq$ is non-trivial and reflexive.
- Completeness: $f \preceq g$ or $f \succeq g$.
- Transitivity: if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.
- Independence: $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.
- Continuity: $\{(\alpha,\beta) \in [0,1]^2 | \; f_\alpha h \preceq g_\beta h\}$ is closed.
- Monotonicity: if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Incomplete preferences

Axioms

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Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

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Incomplete preferences

Axioms

Standard unanimity model (Bewley):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$ whenever f, g are constant acts.

Transitivity if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{(\alpha, \beta) \in [0, 1]^2 | \ f_\alpha h \preceq g_\beta h\}$ is closed.

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Incomplete preferences

Axioms

Confidence-based incomplete preference model:

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Consistency when the stakes decrease, one cannot suspend (determinate) preferences.

Incomplete preferences

Representation theorem

Theorem

$\preceq$ satisfies axioms above

$\Leftrightarrow$ *there exists affine $u : C \to \Re$, Ξ and $D : \mathcal{A}^2 \to \Xi$ such that, for all $f, g \in \mathcal{A}$, $f \preceq g$ iff*

$$\sum_{s \in S} \mathbb{E}_p(u(f)) \leqslant \sum_{s \in S} \mathbb{E}_p(u(g)) \quad \forall p \in D((f, g))$$

Furthermore u is unique up to positive affine transformation, and Ξ and D are unique.

General moral

Expected utility

Remove "precision" ↓

"Set of priors" rule

Remove stakes-independence ↓

Confidence rule

Conclusion

The basic issue between the confidence family and fixed set of priors (aka imprecise probability) models:

- is stakes-independence a rationality constraint?

Dutch Books

The standard argument (approximately):

Conditions on bets $\Leftrightarrow$ betting quotients are probabilities

Where:

Bet on A with stakes S yields €S if A and €0 if not A.

Betting quotient $q(A)$ value such that you are indifferent between buying and selling the bet at stakes S for €$q(A)S$.

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In prevision language:

RV X with stakes S_X E.g. $S_X = \max_{\omega \in \Omega} |X(\omega)|$

$\frac{\underline{P}(X)}{S_X}$ for lower prevision $\underline{P}$

Dutch Books

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Conditions on bets $\Leftrightarrow$ betting quotients are probabilities

Assumptions:

For any bets on events A with stakes S:

- €$q(A)S$ is the price at which you are indifferent between buying and selling the bet
- €$q(A)S$ is the buying / selling price for all stakes S

Buy-sell coincidence $\underline{P}(X) = -\underline{P}(-X)$

Stakes Independence $\frac{\underline{P}(\frac{S}{S_X}X)}{S} = \frac{\underline{P}(\frac{T}{S_X}X)}{T}$

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Removing it gives:

Conditions on bets $\Leftrightarrow$ buying / selling prices are minimal / maximal probabilities of fixed set of priors

Assumptions:

For any bets on events A with stakes S:

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Dutch Books

Which gives:

Conditions on bets $\Leftrightarrow$ buying / selling prices are minimal / maximal probabilities of a confidence ranking

Assumptions:

For any bets on events A with stakes S:

- you have a buying price € $\underline{q_S}(A)S$ and a selling price €$\overline{q_S}(A)S$
- quotients $\underline{q_S}(A)$, $\overline{q_S}(A)$ may depend on stakes
- whatever price you accept for high stakes, you will accept for lower stakes

Stakes Dependence $T \leqslant S \Rightarrow \frac{\underline{P}(\frac{S}{S_X}X)}{S} \leqslant \frac{\underline{P}(\frac{T}{S_X}X)}{T}$

Stakes Independence, in the language of previsions

Coherent lower previsions:

Constant additivity $\underline{P}(X + c1) = \underline{P}(X) + c$

Positive homogeneity $\underline{P}(\alpha X) = \alpha\underline{P}(X)$

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- Both lack normative justification

→ Confidence decision maxim justifies potential violation of both

General moral

Expected utility

Remove "precision"

"Set of priors" rule

Remove stakes-independence

Confidence rule

Conclusion

The basic issue between the confidence family and fixed set of priors models:

- is stakes-independence a rationality constraint?

▸ Enough

Desiderata

✓ it has acceptable choice-theoretical consequences

2. it is conceptually clear about the roles of different mental attitudes

And of course:

✓ it can fruitfully deal with “severe uncertainty” situations such as those above.

Separation of beliefs and desires: Maxmin EU
$(\min_{p \in \mathcal{C}} \mathbb{E}_p u(f(s)))$

Elements of the model:

- Utility function
- Set of priors

Separation of beliefs and desires: Maxmin EU

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Elements of the model:

- Utility function = Desires over outcomes
- Set of priors = Beliefs?

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There is a natural comparison of attitude to uncertainty

- DM 1 is more averse to uncertainty than DM 2 if $\forall f$, constant acts c, $f \succeq_1 c \Rightarrow f \succeq_2 c$.

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For DMs with the same u, DM 1 is more averse to uncertainty iff $\mathcal{C}_1 \supseteq \mathcal{C}_2$. (Ghirardato and Marinacci, 2002)

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Policy example

Policy maker 1's set of priors, $\mathcal{C}_1 \supseteq \mathcal{C}_2$, Policy maker 2's set.

- Does 2 have further information / beliefs?
- Or is he just less cautious?

Separation of beliefs and desires: Maxmin EU

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Policy maker 1's set of priors, $\mathcal{C}_1 \supseteq \mathcal{C}_2$, Policy maker 2's set.

- Does 2 have further information / beliefs?
- Or is he just less cautious?

Maximin EU can't decide the question . . . or even properly represent the possibilities.

Separation of beliefs and desires: Confidence

Elements of the model:

- Utility function
- Confidence ranking
- Cautiousness coefficient
- Decision rule

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And there is a natural comparison of attitude to uncertainty

that corresponds precisely to differences in the cautiousness coefficient.

For DMs with the same u and Ξ

- 1 is more averse to uncertainty

$\Leftrightarrow$ $D_1(f) \supseteq D_2(f)$ for all acts f.

Separation of beliefs and desires: Confidence

Elements of the model:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence
- Decision rule: Unanimity

There is a natural comparison of decisiveness

- DM 1 is more indecisive than DM 2 if $\forall f, g \; f \succeq_1 g \;\Rightarrow f \succeq_2 g$.

Separation of beliefs and desires: Confidence

Elements of the model:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence
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that corresponds precisely to differences in the cautiousness coefficient

For DMs with the same u and Ξ

1 is more indecisive

$\Leftrightarrow$ $D_1((f,g)) \supseteq D_2((f,g))$ for all pairs f and g.

Separation of beliefs and desires

Elements of the model:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence
- Decision rule

Conclusion There is a clean separation between beliefs and desires (attitudes to outcomes and to choosing in the absence of confidence).

Comparison Maxmin EU, unanimity preferences, as well as many other "non-EU" models of decision making, do not exhibit such a separation.

In summary

Belief representation & Decision

1. it has acceptable choice-theoretical consequences
2. it involve a neat separation of beliefs and tastes

And of course:

3. it can fruitfully deal with “severe uncertainty” situations such as those above.

In summary

Belief representation & Decision

1. it has acceptable choice-theoretical consequences
2. it involve a neat separation of beliefs and tastes

And of course:

3. it can fruitfully deal with “severe uncertainty” situations such as those above.

Moreover

4. it is involves an ordinal notion of confidence (useful wrt tractability)

- It is the only model (I am aware of) with 2, 3 and 4.

Ambiguity models Uncertainty Attitudes Climate policy

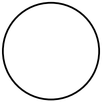
Ambiguity models

Decision models can be categorised by their ‘belief’ component

Model	Belief repn
Maxmin EU & al	Set of priors

Maxmin-EU

$\min_{p \in \mathcal{C}} \mathbb{E}_p(u(f))$



Δ

Ambiguity models

Decision models can be categorised by their 'belief' component

Model	Belief repn
Maxmin EU & al	Set of priors
Multiplier / Variational	Function on prob. space

Multiplier / Variational

$\min_{p\in\Delta}\left(\mathbb{E}_p(u(f)) + c(p)\right)$

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Multiplier / Variational

$\min_{p\in\Delta}\left(\mathbb{E}_p(u(f)) + c(p)\right)$

- ? normative plausibility
- ✗ belief-taste separation
- ✗ ordinal at the 2nd-order level

Ambiguity models

Decision models can be categorised by their 'belief' component

Model	Belief repn
Maxmin EU & al	Set of priors
Multiplier / Variational	Function on prob. space
Smooth	2nd-order prob.

Smooth $\mathbb{E}_{\mu}\phi\left(\mathbb{E}_p(u(f))\right)$

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- ✗? normative plausibility
- ✓? belief-taste separation
- ✗ ordinal at the 2nd-order level

Ambiguity models

Decision models can be categorised by their ‘belief’ component

Model	Belief repn
Maxmin EU & al	Set of priors
Confidence	Confidence Ranking
Multiplier / Variational	Function on prob. space
Smooth	2nd-order prob.

Uncertainty attitudes

- Utility function : Desires
- Confidence ranking : Beliefs and confidence in beliefs
- Cautiousness coefficient : Uncertainty Attitudes
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Example: α-maxmin EU

$$\alpha \min_{p \in D(f)} \mathbb{E}_p(u(f)) + (1-\alpha) \max_{p \in D(f)} \mathbb{E}_p(u(f))$$

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  - Value assigned to the “Pragmatic”

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- α: attitude to uncertainty / imprecision.
  - Prudence; concern for robustness in decision
  - Value assigned to the “Pragmatic”
- Cautiousness coefficient:
  - How willing to risk big decisions on fragile beliefs
  - Value assigned to the “Epistemic”

Uncertainty attitudes

- Utility function : Desires
- Confidence ranking : Beliefs and confidence in beliefs
- Cautiousness coefficient : Uncertainty Attitudes
- Decision rule : Uncertainty Attitudes

Moral

Decision under (severe) uncertainty involves Uncertainty Attitudes:

- Value judgements
- Multi-dimensional

Plan

Back to: Climate Policy & Uncertainty

Aim Overview of issues & a proposal concerning:

How to decide when there is this much uncertainty?

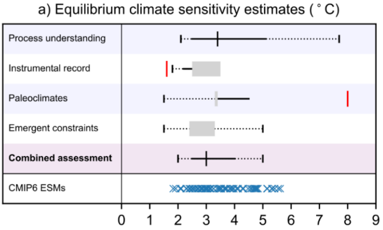


Figure: (Masson-Delmotte et al., 2021, fig TS.16b).

- Best estimate: 3°C
- Likely in range 2.5 to 4°C (high confidence)
- Virtually certain above 1.5°C (high confidence)
- Very likely below 5°C (medium confidence)

(Masson-Delmotte et al., 2021)

Climate Policy, Uncertainty & Confidence

IPCC assessments:

- translate directly into confidence rankings
- are difficult to connect to other uncertainty representations

Reporting and modelling
Decision
Conclusion

The confidence framework:

The confidence framework:

- provides a (hitherto absent) “logic of confidence”

e.g. ECS likely in 2.5–4°C (high conf.) *implies* that ECS *unlikely* greater than 6°C (high conf.)

e.g. the ECS statements are consistent

The confidence framework:

- provides a (hitherto absent) “logic of confidence”
- provides the tools for systematic propogation to “downstream” modelling

e.g. standard imprecise prob. tools *at each level*

and “lifting” rules: if held with high confidence then held with (at least) medium confidence

‘Combining probability with qualitative degree-of-certainty metrics in assessment” with R. Bradley and C. Helgeson

The confidence framework:

- provides a (hitherto absent) “logic of confidence”
- provides the tools for systematic propogation to “downstream” modelling
- provides recommendations for future reporting practices

e.g. report confidence in a likelihood assessment

not confidence before likelihood assessment

The confidence framework:

The confidence framework:

- shows how IPCC reports can be used to guide decision making
- Belief / value separation:
  - policy makers choose the appropriate confidence level for the decision at hand
  - they use the assessments made with that much confidence to decide
  - they select the appropriate ‘pragmatic’ uncertainty attitude (robustness etc.)

The confidence framework:

- shows how IPCC reports can be used to guide decision making
- is a pragmatic vindication of IPCC practices

- Solid normative credentials for the confidence model

→ justification of the IPCC framework

The confidence framework:

- shows how IPCC reports can be used to guide decision making
- is a pragmatic vindication of IPCC practices
- provides practical recommendations for the future use of the language

e.g. policy maker implication in the choice of confidence level to report

e.g. reporting at multiple levels

Confidence representations

Given a set of (IPCC) confidence statements:

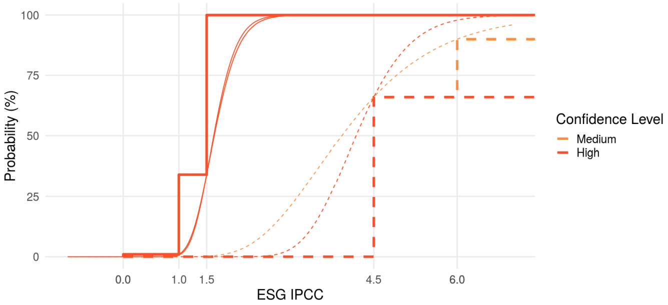
$\rightarrow$ present the nested family of distributions satisfying them.

Confidence calibration Conclusion

Confidence representations

Given a set of (IPCC) confidence statements:

→ present the nested family of distributions satisfying them.



Confidence calibration

Conclusion

Confidence representations

Given a set of (IPCC) confidence statements:

→ present the nested family of distributions satisfying them.

Web tool does this:

- uses the logic just discussed
- detects inconsistencies
- traces p-boxes for each confidence level

Also:

- Elicitation of confidence in beliefs

`http://confidence.hec.fr/app/`

▸ Confidence calibration ▸ Conclusion

Plan

Probabilistic belief aggregation

i (honest, well-intentioned) experts; 0 group.

Only differ in beliefs (same u^i).

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Linear opinion pooling For all events E:

$$p^0(E) = \sum_{i=1}^{n} w^i p^i(E)$$

for weights w^i.

$$\Uparrow$$

Pareto $f \succeq^i g$ for all $i \Rightarrow f \succeq^0 g$

- Respect (issue-level) consensus

(Mongin, 1995)

Challenges

Example

Challenges

Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L)p^R(E)$$

Challenges
Example

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Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L) p^R(E)$$

Challenges
Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L)p^R(E)$$

Spurious Unanimity

- Why respect spurious consensus?

(Mongin, 2016; Mongin and Pivato, 2020; Dietrich, 2021; Bommier et al., 2021)

Challenges
Example

Probability certain interest rate rise has limited effect on:

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Laura	0.9	0.1	0.09
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$$p^0(E) = w^L p^L(E) + (1 - w^L)p^R(E)$$

Spurious Unanimity

- Why respect spurious consensus?

Diverse (intra-agent) expertise

- Why is Laura's judgement on Labour treated the same as her judgement on Real-estate?

(Genest and Zidek, 1986; French, 1985)

Challenges
Summary

Desiderata A belief aggregation procedure that:

1. respects the right consensus(es)
   - avoiding spurious unanimities
2. can do justice to varying expertise

Proposal

Insights

Issue-level consensus

Spuriousness

Desideratum A belief aggregation procedure that:

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↓

Evidence, reasons, information

$p^i(E)$ does not exhaust the elements of belief states pertaining to event E relevant for aggregation ...

Proposal

Insights

Issue-level consensus

Spuriousness

↓

Evidence, reasons, information

↓

Confidence

$p^i(E)$ does not exhaust the elements of belief states pertaining to event E relevant for aggregation . . .

Confidence in beliefs: Hill, 2013; **?**; **?**; Bradley, 2017

but also Klibanoff et al., 2005a; Maccheroni et al., 2006a; Hansen and Sargent, 2008; Chateauneuf and Faro, 2009a

Proposal

Insights

Issue-level consensus

Spuriousness

↓

Evidence, reasons, information

↓

Confidence

Desideratum A belief aggregation procedure that:

1. respects the right consensus(es)
    - avoiding spurious unanimities

Proposal

Insights

Issue-level consensus

↓

Corpus-level consensus

Spuriousness

↓

Evidence, reasons, information

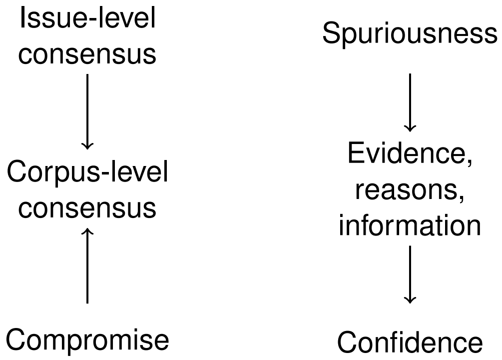
↓

Confidence

Corpus: (coherent) set of probability judgements $\equiv$ set of priors.

Proposal

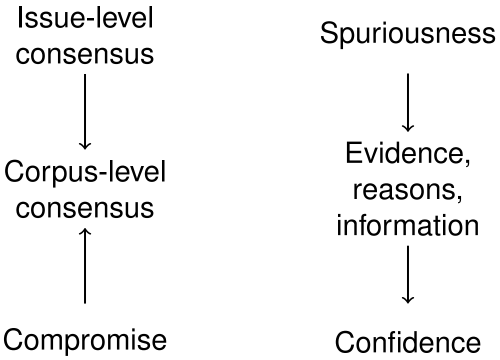
Insights



Corpus-level consensus: Everyone is willing to ‘leave off the table’ or compromise any potential disagreement.

Proposal

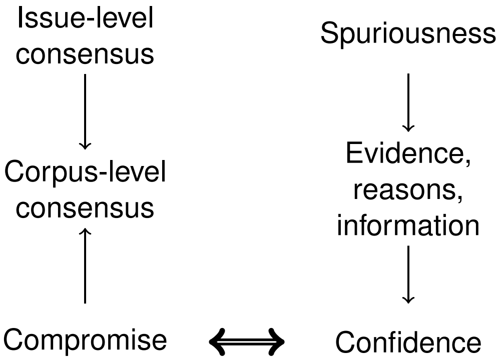
Insights



What compromises are agents willing to make?

Proposal

Insights



Confidence and Compromise

The more confident an individual is in a belief, the less willing she is to compromise on it.

In aggregation:

Respect corpus-level consensus

where

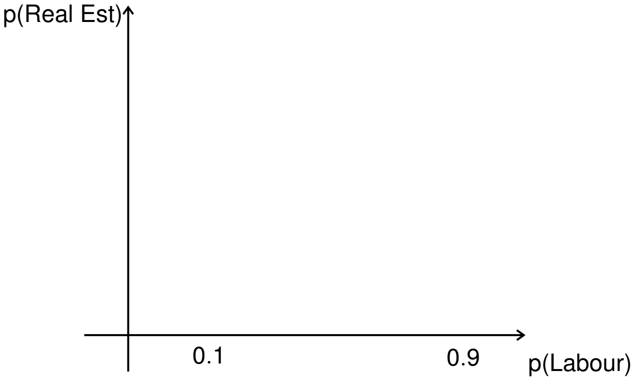
The more confident an individual is in a belief,
the less willing she is to compromise on it.

Aim

- Develop such an aggregation rule
- Spurious unanimity, expertise ... and beyond.

Plan

Confidence in beliefs

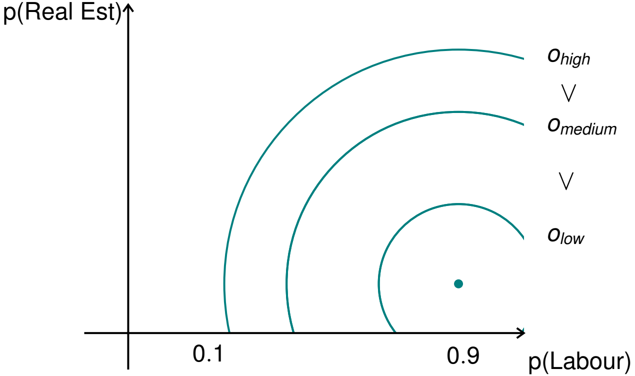


Preliminaries

Δ probability measures (over states Ω)

$(O, >)$ confidence levels

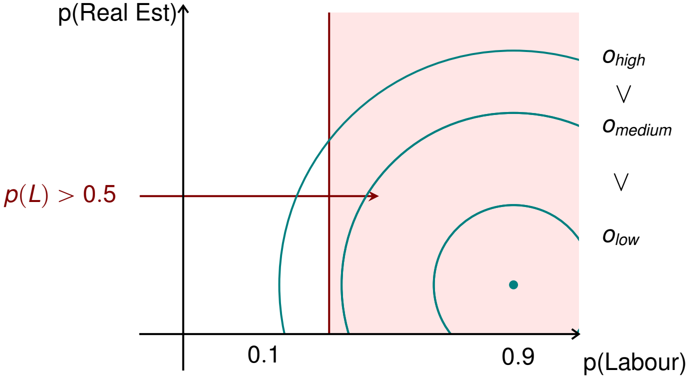
Confidence in beliefs



Confidence ranking: Increasing $c^i : O \to 2^{\Delta} \setminus \varnothing$.

(Hill, 2013; **?**; Manski, 2013; Bradley, 2017)

Confidence in beliefs



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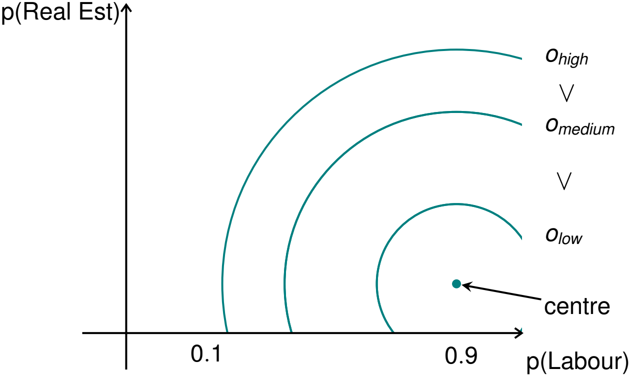

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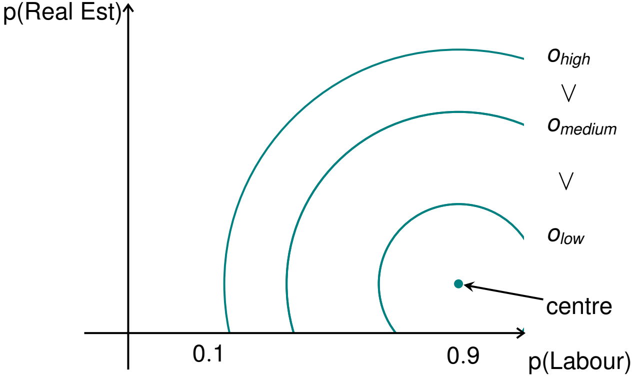


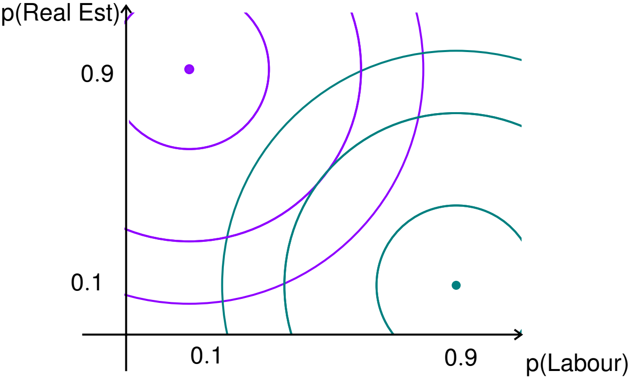
Confidence ranking: Increasing $c^i : O \rightarrow 2^{\Delta} \backslash \emptyset$.

Implausibility fn $\iota : \Delta \rightarrow O \cup \emptyset$

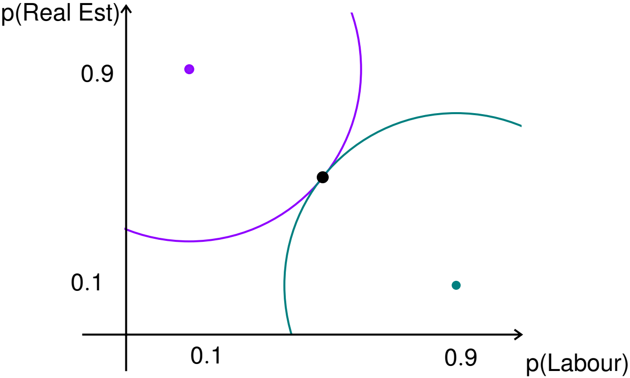
(Hill, 2013; **?**; Manski, 2013; Bradley, 2017)[a]

[a] Reduced form for Klibanoff et al., 2005a; Maccheroni et al., 2006a; Hansen and Sargent, 2008; Chateauneuf and Faro, 2009a . . .

Consensus



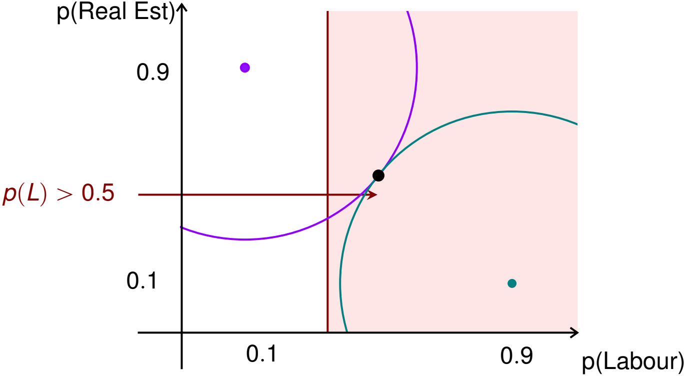
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

$$\bigcap_i c^i(o)$$

Consensus



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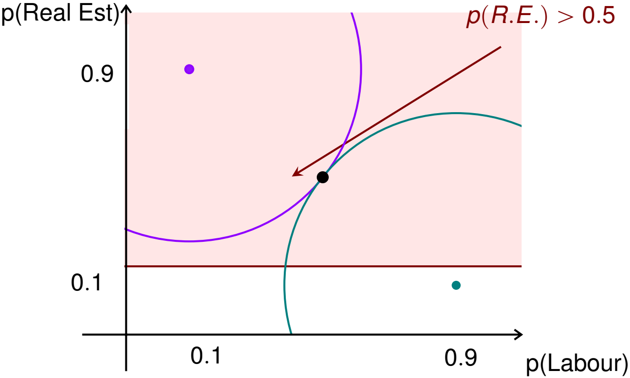

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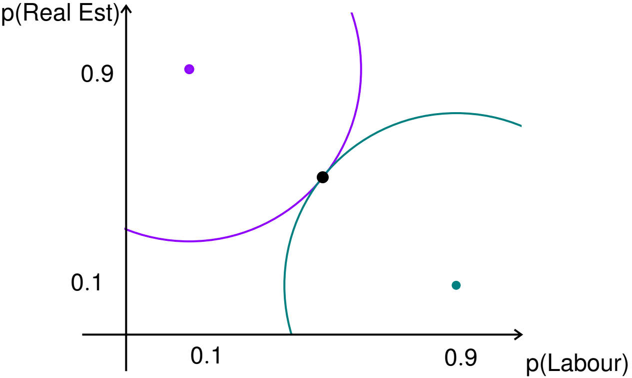

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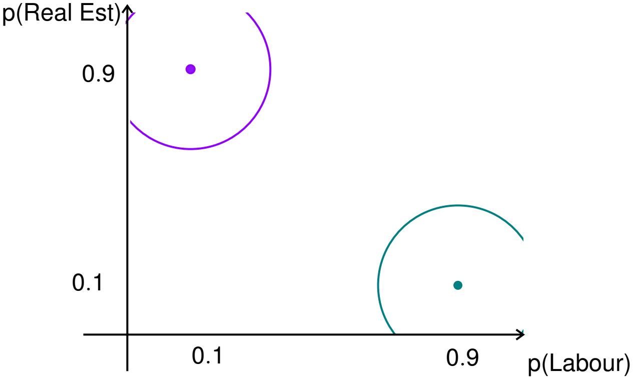
Maxim If more confident in a belief, less willing to compromise it.

Consensus



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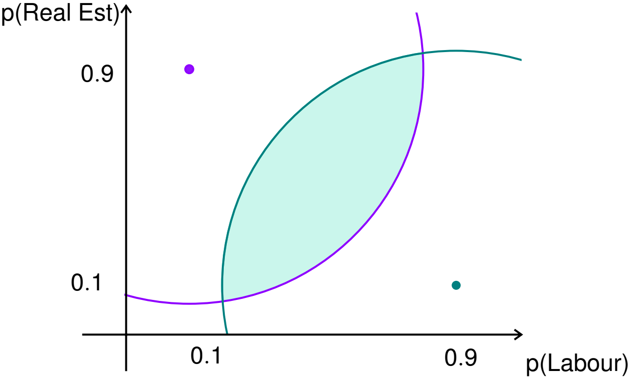

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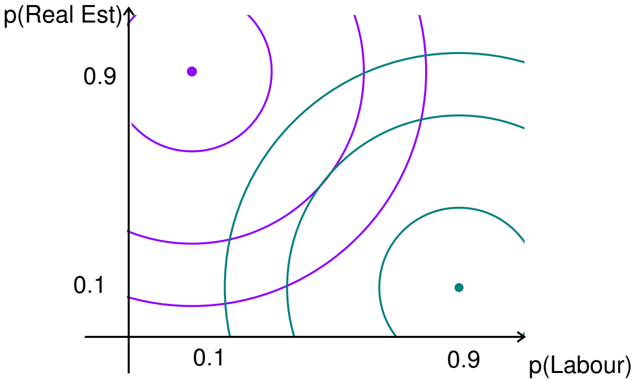
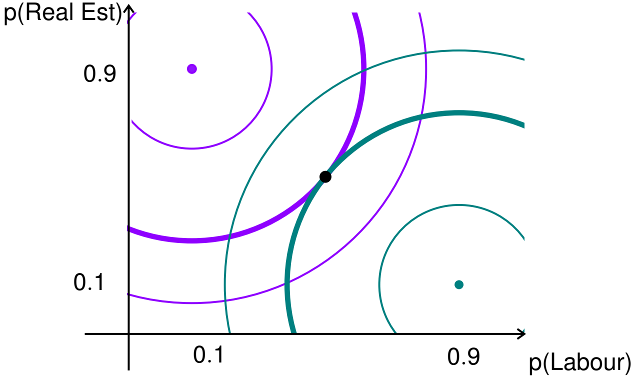
Confidence aggregation



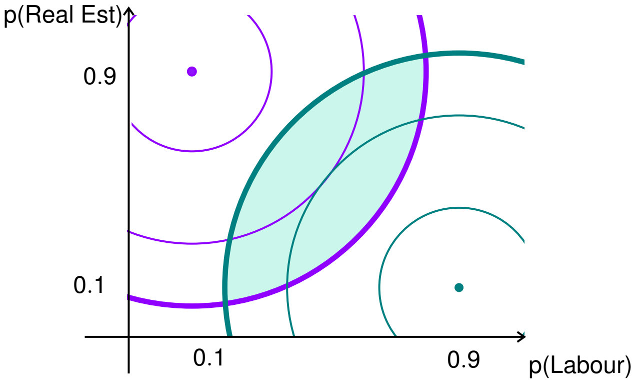
Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i)}_{\textit{Consensus}}$$



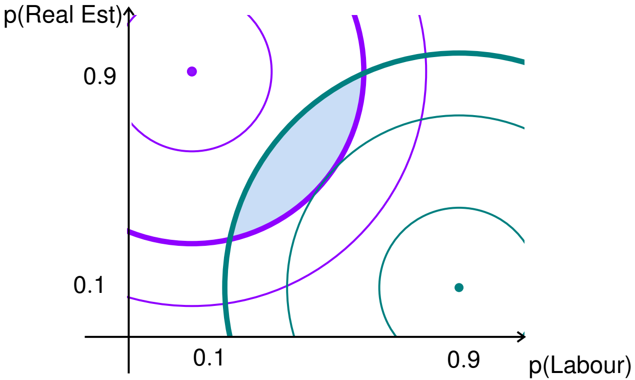
Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i')}_{Consensus}$$



Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i'')}_{\textit{Consensus}}$$



Confidence aggregation

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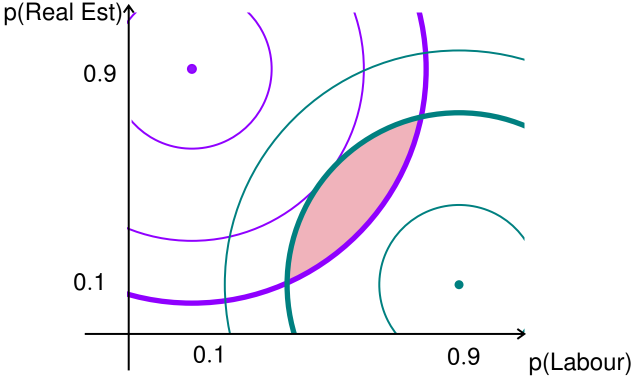

Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i)}_{Consensus}$$

$\otimes : O^n \to O$: confidence level aggregator.

- $\otimes o$: group confidence in consensus judgements in o
- monotonic

Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i)}_{Consensus}$$

$\otimes : O^n \rightarrow O$: confidence level aggregator.

- $\otimes o$: group confidence in consensus judgements in o
- monotonic

E.g.

Maximum agg. $\otimes o = \max \{o_i\}$

Minimum agg. $\otimes o = \min \{o_i\}$

Average agg. $\otimes o = \sum \frac{1}{n} o_i + \chi$

Confidence aggregation

$$F_{\otimes}(c^1, \ldots, c^n)(o) = \bigcup_{o : \otimes o \leqslant o} \underbrace{\bigcap_i c^i(o_i)}_{Consensus}$$

for o with $\bigcap_i c^i(o_i) \neq \emptyset$

Group judgement: held in all consensuses with that level of confidence.

Confidence aggregation

Consensus-preserving confidence aggregation:

$$F_{\otimes}(c^1, \ldots, c^n)(o) = \bigcup_{\mathbf{o}:\otimes\mathbf{o} \leqslant o} \bigcap_i c^i(o_i)$$

The more individual confidence there is in a consensus judgement, the more confidence the group has in it.

Confidence aggregation

Consensus-preserving confidence aggregation:

$$F_{\otimes}(c^1, \ldots, c^n)(o) = \bigcup_{o : \otimes o \leqslant o} \bigcap_i c^i(o_i)$$

Equivalently:

$$F_{\otimes}(\iota^1, \ldots, \iota^n)(p) = \otimes(\iota^1(p), \ldots, \iota^n(p))$$

So

$$\mathrm{Centre}_{F_{\otimes}(c^1,\ldots,c^n)} = \arg\min_{p \in \Delta} \otimes(\iota^1(p), \ldots, \iota^n(p))$$

$$\overset{avge}{=} \otimes \arg\min_{p \in \Delta} \sum_{i=1}^{n} \iota^i(p)$$

Plan

Pooling & Confidence

Probabilities p^i

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i \rho(q, p^i) \leqslant o \right\}$$

ρ: distance[a]

E.g.

Euclidean $\rho(q,p) = \sum_{\omega \in \Omega} (q(\omega) - p(\omega))^2$

Relative Entr. $\rho(q,p) = R(q \| p)$

Reverse Rel. Entr. $\rho(q,p) = R(p \| q)$

[a] lower semicts; $\rho(q,p) = 0 \Leftrightarrow p = q$.

Pooling & Confidence

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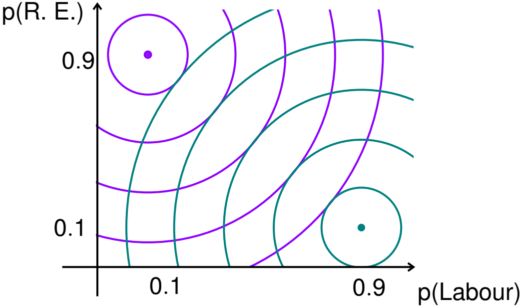

ρ: Euclidean

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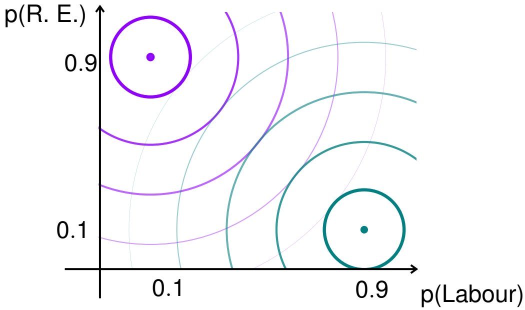

$w^L = w^R$

ρ: Euclidean

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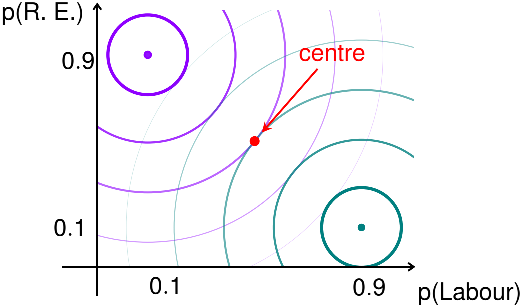

Confidence aggregation
$w^L = w^R$

ρ: Euclidean

Average $\otimes$

Pooling & Confidence

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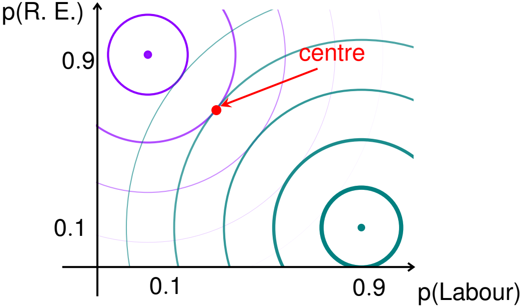

Confidence aggregation
$w^L < w^R$

ρ: Euclidean

Average $\otimes$

Pooling & Confidence

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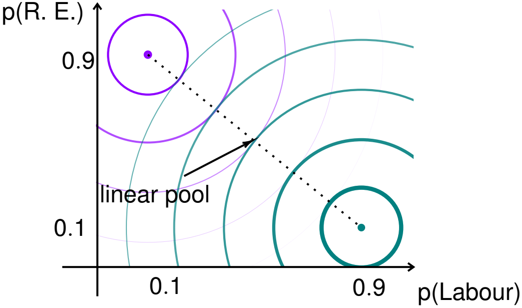

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Average $\otimes$

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i \rho(q, p^i) \leqslant o \right\}$$

Theorem

Centre of confidence aggregation = result of pooling rule

Generating distance	*Pooling rule*
Euclidean	*Linear*
Relative Entropy	*Geometric*
Reverse Rel. Entr.	*Linear*

with weights $\frac{w^i}{\sum_{i=1}^{n} w^i}$.

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i \rho(q, p^i) \leqslant o \right\}$$

Theorem

Centre of confidence aggregation = result of pooling rule

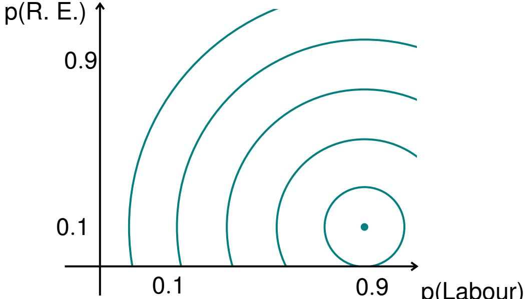
Moral Linear pooling =

- special case of confidence aggregation
- corresponding to assumptions on individuals' confidence.

Expertise

Euclidean generated confidence ranking:

$$c^L(o) = \left\{ q \in \Delta : w^L \sum_{s \in S'} (q(s) - p^L(s))^2 \leqslant o \right\}$$



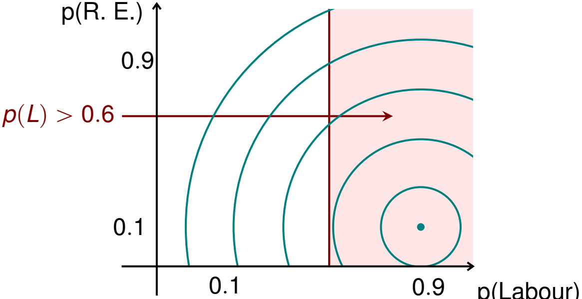
Expertise

Euclidean generated confidence ranking:

$$c^L(o) = \left\{ q \in \Delta : w^L \sum_{s \in S'} (q(s) - p^L(s))^2 \leqslant o \right\}$$

Fact Euclidean generated distance assumes

- divergence from p^i on Labour $\equiv$ divergence on R. E.



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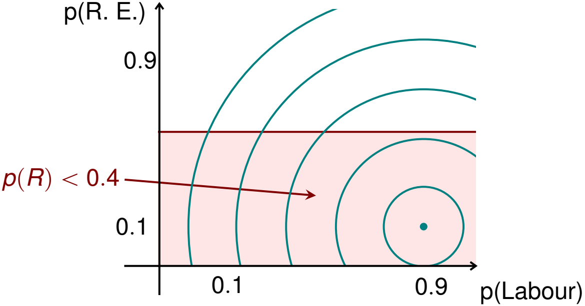

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Fact Euclidean generated distance assumes

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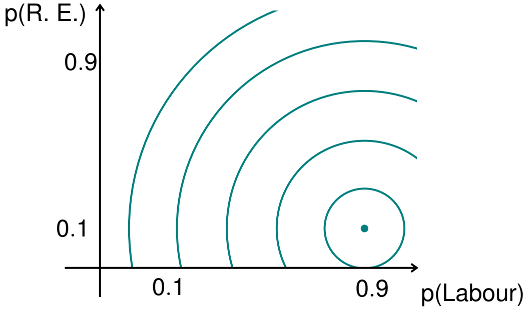
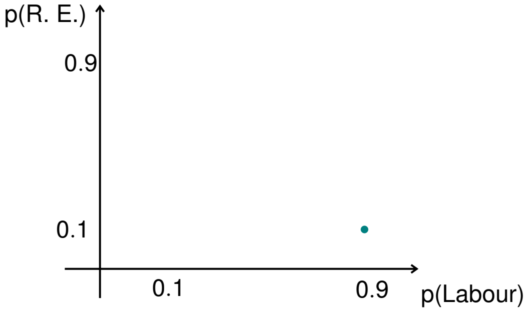
→ same confidence on all issues!



Expertise

$$c^L(o) = \left\{ q \in \Delta : \begin{array}{l} w^L_L(q(L) - p^L(L))^2 \\ +w^L_R(q(R) - p^L(R))^2 \end{array} \leqslant o \right\}$$



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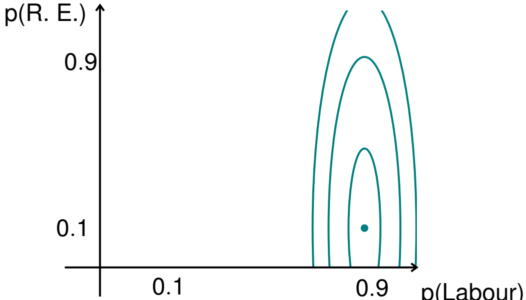
$w^L_L > w^L_R$



Expertise

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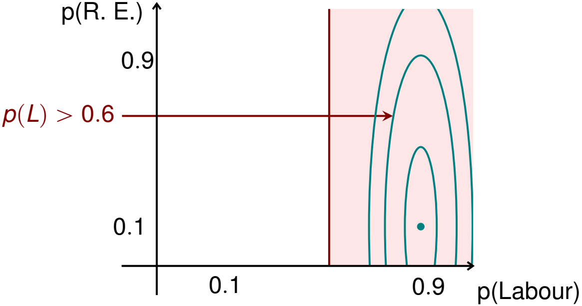
Fact $w^L_L > w^L_R$: more confident in Labour judgements



Expertise

$$c^L(o) = \left\{ q \in \Delta : \begin{array}{l} w^L_L(q(L) - p^L(L))^2 \\ + w^L_R(q(R) - p^L(R))^2 \end{array} \leqslant o \right\}$$

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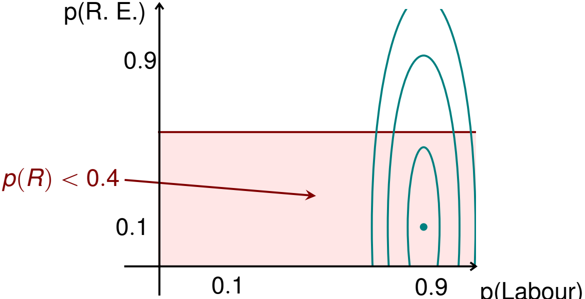
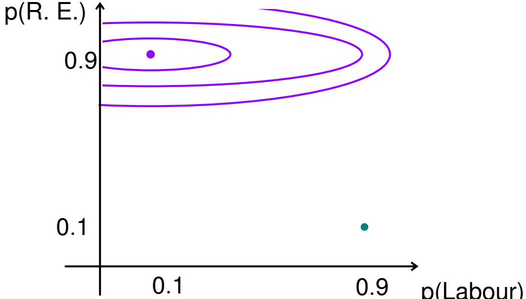


Expertise

$$c^R(o) = \left\{ q \in \Delta : \begin{array}{l} w^R_L(q(L) - p^L(L))^2 \\ + w^R_R(q(R) - p^L(R))^2 \end{array} \leqslant o \right\}$$

$w^R_L < w^R_R$: more confident in Real-Estate judgements



Expertise

Confidence: rich enough to capture diverse expertise.

In general: (w^i, d, p)-generated confidence ranking:

$$c^i(o) = \left\{ q \in \Delta : \sum_{j=1}^{m} w^i_j d(q|_{\mathcal{P}_j}, p|_{\mathcal{P}_j}) \leqslant o \right\}$$

where:

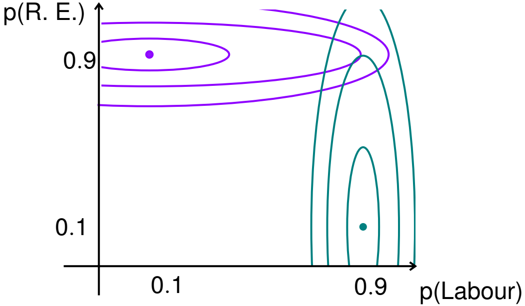
- $\mathcal{P}_j$ Issues: partitions of Ω
- w^i vector of weights
- d distance, for each partition

Often, can rewrite e.g.

$$c^i(o) = \{q \in \Delta : (q - p^i)^T D^i (q - p^i) \leqslant o\}$$

Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.



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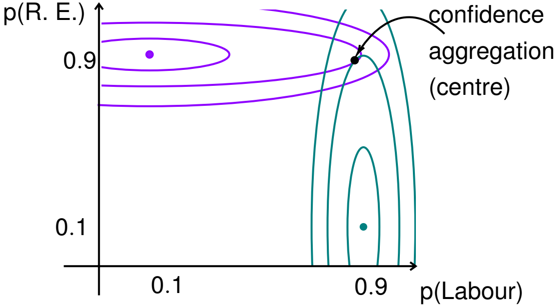
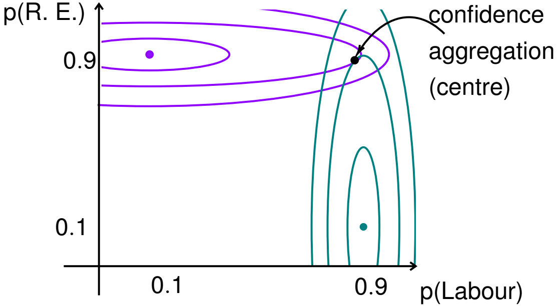


Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.

Confidence aggregation:

- Does justice to varying expertise (Desideratum 2)

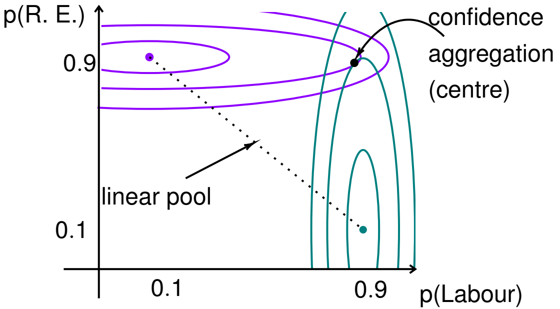


Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.

Confidence aggregation:

- Does justice to varying expertise (Desideratum 2)
- Does not necessarily respect spurious unanimities (Desideratum 1)



Aggregation and Expertise

More generally:

Theorem

Confidence aggregation (w^i, d, p^i)*-generated confidence rankings:*

$$Centre =_{p \in \Delta} \sum_{i=1}^{n} \sum_{j=1}^{m} w_j^i d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

When d convex and the issues sufficiently rich: single probability.

- Copes with within-person expertise diversity & spurious unanimity
- Some cases: ${}_{Aq \leqslant r} \sum_{i=L,R} (q - p^i)^T D^i (q - p^i)$
- Always non-empty (even when issue-dependency)

Aggregation and Expertise

Generates a new probability aggregation rule:

Expert-sensitive pooling

$$F^{d}_{\mathcal{P}_1,\ldots,\mathcal{P}_m}(p^1,\ldots,p^n) =_{p\in\Delta} \sum_{i=1}^{n}\sum_{j=1}^{m} w^i_j d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

for convex d and rich $\{\mathcal{P}_j\}$.

- Copes with within-person expertise diversity & spurious unanimity
- Tractable cases
- Well-defined

Aggregation and Expertise

Generates a new probability aggregation rule:

Expert-sensitive pooling

$$F^d_{\mathcal{P}_1,\ldots,\mathcal{P}_m}(p^1,\ldots,p^n) =_{p\in\Delta} \sum_{i=1}^{n}\sum_{j=1}^{m} w^i_j d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

for convex d and rich $\{\mathcal{P}_j\}$.

- Copes with within-person expertise diversity & spurious unanimity
- Tractable cases
- Well-defined
- Resolves a long-standing challenge (Genest and Zidek, 1986; French, 1985).

Conclusion

Confidence in beliefs has a role in:

Rational Decision

- Formal model of confidence in beliefs and decision framework
- Account with attractive conceptual and choice-theoretic properties
- Belief-value separation
- Consequences for high-stakes decision making

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Thank you.

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Further details (all available on the website above):

- Confidence in Beliefs and Rational Decision Making, *Economics and Philosophy*, 32, 2019.
- Confidence and Decision, *Games and Economic Behavior*, 82, 2013.
- Incomplete Preferences and Confidence, *Journal of Mathematical Economics*, 65, 2016.
- Climate Change Assessments: Confidence, Probability and Decision, *Philosophy of Science*, 84, 2017 (with R. Bradley, C. Helgeson).
- Combining probability with qualitative degree-of-certainty metrics in assessment, *Climatic Change* 149, 2018 (with R. Bradley, C. Helgeson)
- Confidence in belief, weight of evidence and uncertainty reporting. *Proceedings of Machine Learning Research*, 103, 2019.
- Updating Confidence in Beliefs, *Journal of Economic Theory*, 2022.
- Confidence, consensus and aggregation, *HEC Working Paper*, 2024.

Confidence Elicitation Web Tool http://confidence.hec.fr/app/

Confidence in Beliefs and Rational Policy Making

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References I

Abdellaoui, M., Hill, B., and Akella, S. C. (2024). Disentangling Drivers of Ambiguity Attitudes. *Available at SSRN 4916177*.

Baillon, A. and Placido, L. (2019). Testing constant absolute and relative ambiguity aversion. *Journal of Economic Theory*, 181:309–332. Publisher: Elsevier.

Bommier, A., Fabre, A., Goussebaïle, A., and Heyen, D. (2021). Disagreement aversion. *Available at SSRN 3964182*.

Bradley, R. (2017). *Decision theory with a human face*. Cambridge University Press.

Broome, J. (2012). *Climate matters: Ethics in a warming world (Norton global ethics series)*. WW Norton & Company.

Chateauneuf, A. and Faro, J. H. (2009a). Ambiguity through confidence functions. *J. Math. Econ.*, 45:535–558.

Chateauneuf, A. and Faro, J. H. (2009b). Ambiguity through confidence functions. *J. Math. Econ.*, 45:535–558.

References II

Dietrich, F. (2021). Fully Bayesian Aggregation. *Journal of Economic Theory*, 194:105255. Publisher: Elsevier.

French, S. (1985). Group consensus probability distributions: A critical survey in Bayesian statistics. *Bayesian statistics*, 2.

Genest, C. and Zidek, J. V. (1986). Combining Probability Distributions: A Critique and an Annotated Bibliography. *Statistical Science*, 1(1):114–135.

Ghirardato, P. and Marinacci, M. (2002). Ambiguity made precise: A comparative foundation. *J. Econ. Theory*, 102(2):251 – 289.

Grant, S. and Polak, B. (2013). Mean-dispersion preferences and constant absolute uncertainty aversion. *Journal of Economic Theory*, 148(4):1361–1398.

Hansen, L. P. and Sargent, T. J. (2008). *Robustness*. Princeton university press.

References III

Hill, B. (2013). Confidence and decision. *Games and Economic Behavior*, 82:675–692.

Hill, B. (2019). Confidence in Beliefs and Rational Decision Making. *Economics and Philosophy*.

Hill, B. (2020). Dynamic consistency and ambiguity: A reappraisal. *Games and Economic Behavior*, 120:289–310. Publisher: Elsevier.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005a). A Smooth Model of Decision Making under Ambiguity. *Econometrica*, 73(6):1849–1892.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005b). A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892.

Knight, F. H. (1921). *Risk, Uncertainty, and Profit*. Houghton Mifflin, Boston and New York.

References IV

Maccheroni, F., Marinacci, M., and Rustichini, A. (2006a). Ambiguity Aversion, Robustness, and the Variational Representation of Preferences. *Econometrica*, 74(6):1447–1498.

Maccheroni, F., Marinacci, M., and Rustichini, A. (2006b). Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica*, 74:1447–1498.

Manski, C. F. (2013). *Public Policy in an Uncertain World : Analysis and Decisions*. Harvard University Press, Cambridge, Mass.

Marinacci, M. (2015). Model uncertainty. *Journal of the European Economic Association*, 13(6):1022–1100.

References V

Masson-Delmotte, V., Zhai, P., Pirani, A., Connors, S. L., Péan, C., Berger, S., Caud, N., Chen, Y., Goldfarb, L., Gomis, M. I., Huang, M., Leitzell, K., Lonnoy, E., Matthews, J. B. R., Maycock, T. K., Waterfield, T., Yelekçi, , Yu, R., and Zhou, B., editors (2021). *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press.

Mastrandrea, M. D., Field, C. B., Stocker, T. F., Edenhofer, O., Ebi, K. L., Frame, D. J., Held, H., Kriegler, E., Mach, K. J., and Matschoss, P. R. (2010). Guidance note for lead authors of the IPCC fifth assessment report on consistent treatment of uncertainties. *Intergovernmental Panel on Climate Change (IPCC)*.

Mongin, P. (1995). Consistent Bayesian Aggregation. *Journal of Economic Theory*, 66(2):313–351.

References VI

Mongin, P. (2016). Spurious unanimity and the Pareto principle. *Economics & Philosophy*, 32(3):511–532. Publisher: Cambridge University Press.

Mongin, P. and Pivato, M. (2020). Social preference under twofold uncertainty. *Economic Theory*, 70(3):633–663. Publisher: Springer.

Peirce, C. S. (1878). *Philosophical Writings of Peirce*. Dover, New York.

Savage, L. J. (1954). *The Foundations of Statistics*. Dover, New York. 2nd edn 1971.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, 57(3):571–587.