Conditioning and AGM-like belief change in the Desirability-Indifference framework

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Classical and quantum IP seem very similar, can we generalise this?

Classical IP: **gambles** $f \in \mathscr{G}$ acting on the possibility space $\mathscr{X}$

Quantum IP: **Hermitian operators** $\hat{A} \in \mathscr{H}$ acting on the state space $\mathscr{X}$

Options $u, v \in \mathscr{U}$ → a real vector space

By **accepting** and **rejecting** options, we can model Barbie's uncertainty

$$M := \langle M_{\blacktriangleright} \cup M_{\equiv} \,;\, -M_{\blacktriangleright} \rangle$$

accept, reject, desirable, indifferent

Criteria to be a **rational DI-model**

DI1. $V \subseteq M$;

DI2. $0 \notin M_{\blacktriangleright}$;

DI3. $M_{\blacktriangleright}$ convex cone, $M_{\equiv}$ linear space;

DI4. $M_{\blacktriangleright} + M_{\equiv} \subseteq M_{\blacktriangleright}$.

New information (an **event**) can cause Barbie to modify her beliefs

Events $e \in \mathscr{E}$

E1. $e * (u + \lambda v) = e * u + \lambda e * v$;

E2. $e * u = e * (e * u)$;

E3. $u \succeq 0 \Rightarrow e * u \succeq 0$;

E4. $1_{\mathscr{U}} \succeq 0$ and $1_{\mathscr{U}} * u = u$ and $e * 1_{\mathscr{U}} = e$;

E5. $0_{\mathscr{U}} \in \mathscr{E}$ such that $0_{\mathscr{U}} * u = 0$;

E6. $(\forall u \in \mathscr{U})(e_2 * u = 0 \Rightarrow e_1 * u \not\succ 0)$
$\Rightarrow e_1 \sqsubseteq e_2$;

E7. there's some $\neg e \in \mathscr{E}$ such that

a. $e + \neg e \in \mathscr{E}$ and $(e + \neg e) * u = u$;

b. $e * (\neg e * u) = 0$.

?!

Abstract	Classical	Quantum
events $\mathscr{E} \subseteq \mathscr{U}$	indicators $\mathbb{I}_E$	projections $\hat{P}_{\mathscr{W}}$
$1_{\mathscr{U}}$	$\mathbb{I}_{\mathscr{X}} = 1$	$\hat{P}_{\mathscr{X}} = \hat{I}$
$0_{\mathscr{U}}$	$\mathbb{I}_{\emptyset} = 0$	$\hat{P}_{0} = \hat{0}$
$e_1 \sqsubseteq e_2$	$E_1 \subseteq E_2$	$\mathscr{W}_1 \subseteq \mathscr{W}_2$
$\neg e$	$\mathbb{I}_{\neg e} = 1 - \mathbb{I}_E$	$\hat{P}_{\mathscr{W}^{\perp}} = \hat{I} - \hat{P}_{\mathscr{W}}$

Given an **event** e, how can Barbie update her initial **DI-model** M?

Expansion $\mathrm{E}(M \,|\, M_e)$

$$\begin{cases} \langle M_{\blacktriangleright} \cup \{0\} \,;\, -M_{\blacktriangleright} \rangle & \text{if } e = 1_{\mathscr{U}}; \\ \langle \mathscr{U} \,;\, \mathscr{U} \rangle & \text{otherwise.} \end{cases}$$

These operators are linked to each other:

Levi's identity
$\mathrm{R}(M \,|\, M_e) = \mathrm{E}(\mathrm{C}(M \,|\, M_{\neg e}) \,|\, M_e)$

Harper's identity
$\mathrm{C}(M \,|\, M_e) = M \cap \mathrm{R}(M \,|\, M_{\neg e})$

Revision $\mathrm{R}(M \,|\, M_e)$

$$\langle M_{\blacktriangleright} \,\|\, e \cup I_e \,;\, -M_{\blacktriangleright} \,\|\, e \rangle$$

BR1. $\mathrm{R}(M \,|\, M_e) \in \mathbf{M}_{\mathrm{DI}}(V_e)$;

BR2. $M_e \in \mathrm{R}(M \,|\, M_e)$;

BR3. $\mathrm{R}(M \,|\, M_e) \subseteq \mathrm{E}(M \,|\, M_e)$;

BR4. $\mathrm{E}(M \,|\, M_e) \subseteq \mathrm{R}(M \,|\, M_e)$ if M and M_e are consistent;

BR5. $\mathrm{R}(M \,|\, M_e)$ is inconsistent if and only if M_e is inconsistent;

BR6. $\mathrm{R}(M \,|\, M_e) = \mathrm{R}(M \,|\, \mathrm{cl}_{\mathbf{M}(V_o)}(M_e))$;

BR7. $\mathrm{R}(M \,|\, M_{e_1} \cup M_{e_2}) \subseteq \mathrm{E}(\mathrm{R}(M \,|\, M_{e_1}) \,|\, M_{e_2})$;

BR8. $\mathrm{E}(\mathrm{R}(M \,|\, M_{e_1}) \,|\, M_{e_2}) \subseteq \mathrm{R}(M \,|\, M_{e_1} \cup M_{e_2})$ if $\mathrm{R}(M \,|\, M_{e_1})$ and M_{e_2} are consistent.

$M_{\blacktriangleright} \,\|\, e := \{u \in \mathscr{U} : e * u \in M_{\blacktriangleright}\}$

$I_e := \{u \in \mathscr{U} : e * u = 0\}$

Contraction $\mathrm{C}(M \,|\, M_e)$

$$\langle (M_{\blacktriangleright} \,\|\, \neg e \cap M_{\blacktriangleright}) \cup (I_{\neg e} \cap M_{\blacktriangleright}) \cup \{0\} \,;\, -(M_{\blacktriangleright} \,\|\, \neg e \cap M_{\blacktriangleright}) \rangle$$

BC1. $\mathrm{C}(M \,|\, M_e) \in \mathbf{M}(V_o)$;

BC2. $\mathrm{C}(M \,|\, M_e) \subseteq M$;

BC3. $\mathrm{C}(M \,|\, M_e) = M$ if M and $M_{\neg e}$ are consistent;

BC4. if $M_e \subseteq \mathrm{C}(M \,|\, M_e)$ then $M_{\neg e}$ is inconsistent;

BC5. if $M_e \subseteq \mathrm{C}(M \,|\, M_e)$ then $M \subseteq \mathrm{E}(\mathrm{C}(M \,|\, M_e) \,|\, M_e)$;

BC6. $\mathrm{C}(M \,|\, M_e) = \mathrm{C}(M \,|\, \mathrm{cl}_{\mathbf{M}(V_o)}(M_e))$;

BC7. $\mathrm{C}(M \,|\, M_{e_1}) \cap \mathrm{C}(M \,|\, M_{e_2}) \subseteq \mathrm{C}(M \,|\, M_{e_1} \cup M_{e_2})$;

BC8. $\mathrm{C}(M \,|\, M_{e_1} \cup M_{e_2}) \subseteq \mathrm{C}(M \,|\, M_{e_1})$ if $\mathrm{C}(M \,|\, M_{e_1})$ and $M_{\neg e_1}$ are consistent.