Propagation Algorithms in Credal Networks

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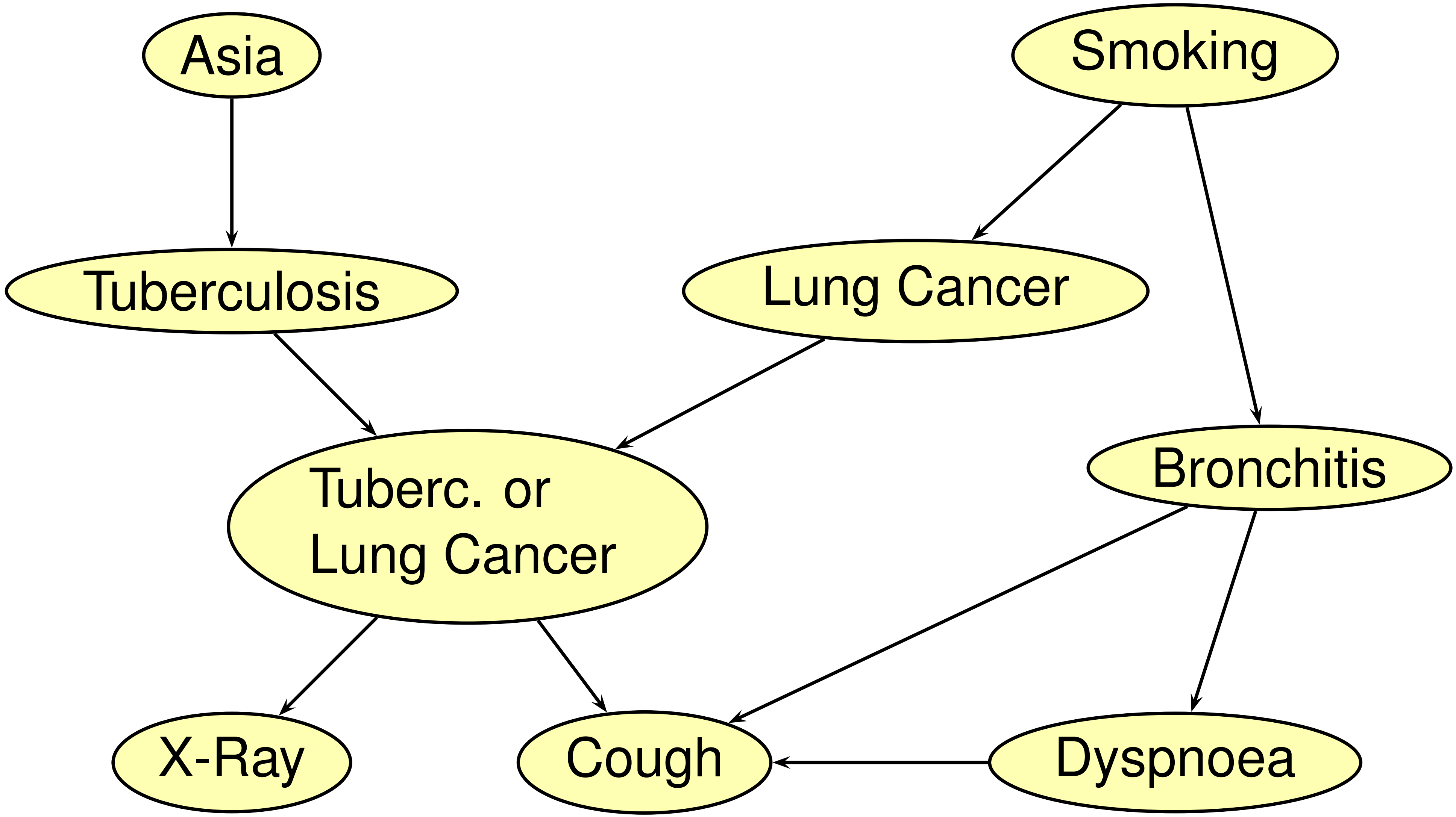
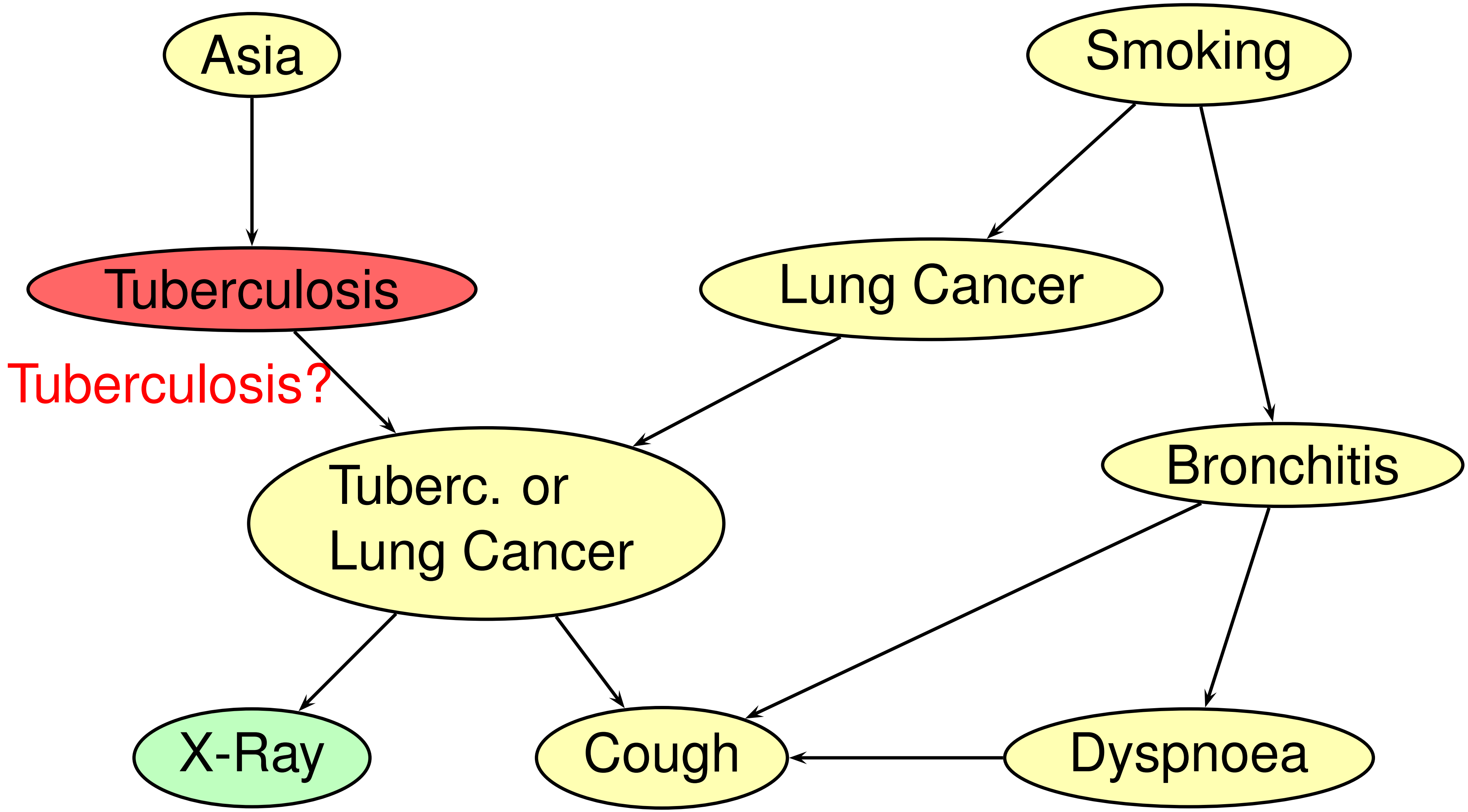
Outline

- Introduction to Propagation in Bayesian Networks.
- Computation with Strong Independence.
- Adding Transparent Variables.
- A Greedy Algorithm.
- Other Approaches.

The Basic Problem: Asia Network

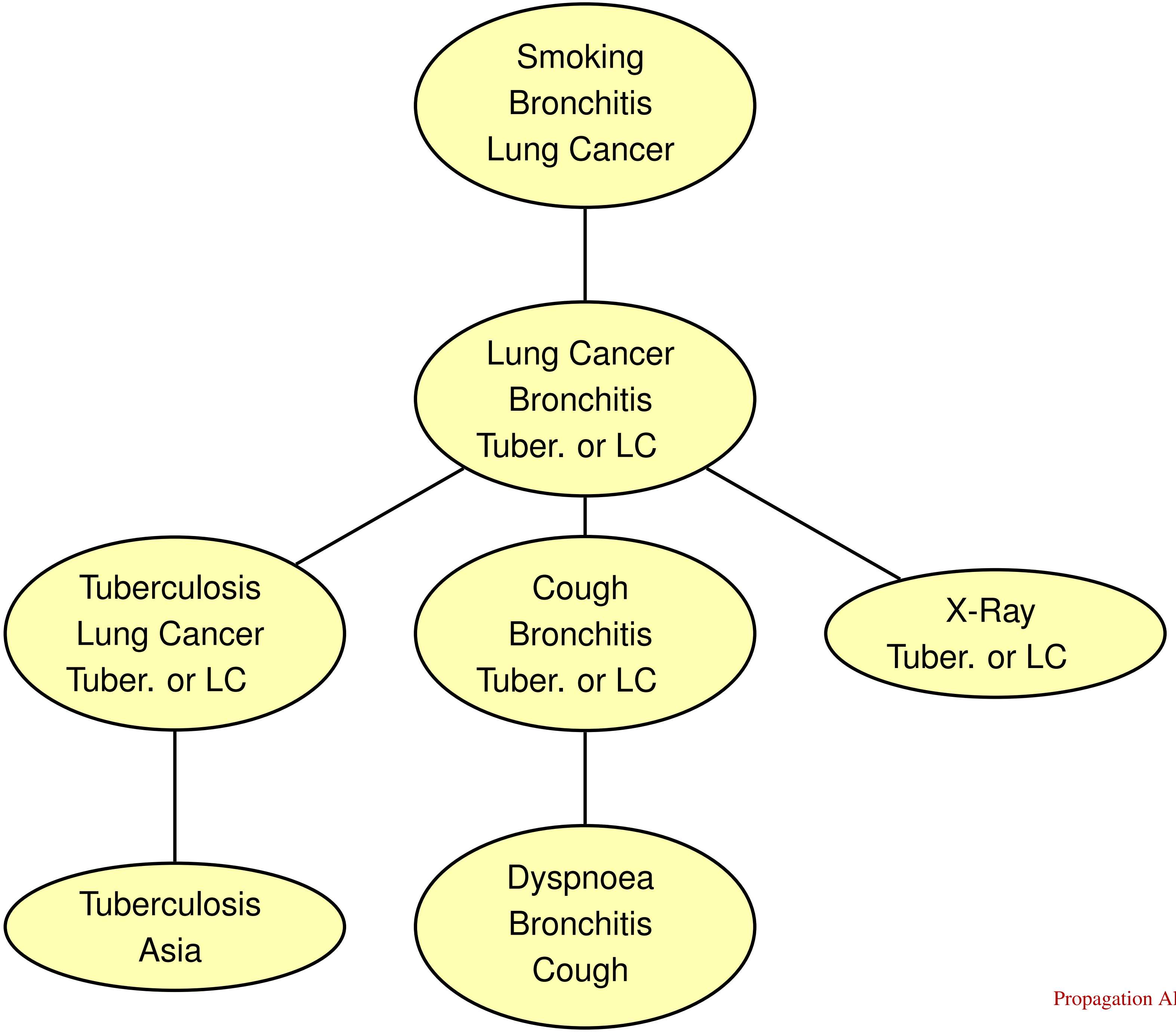


The Basic Problem: Asia Network

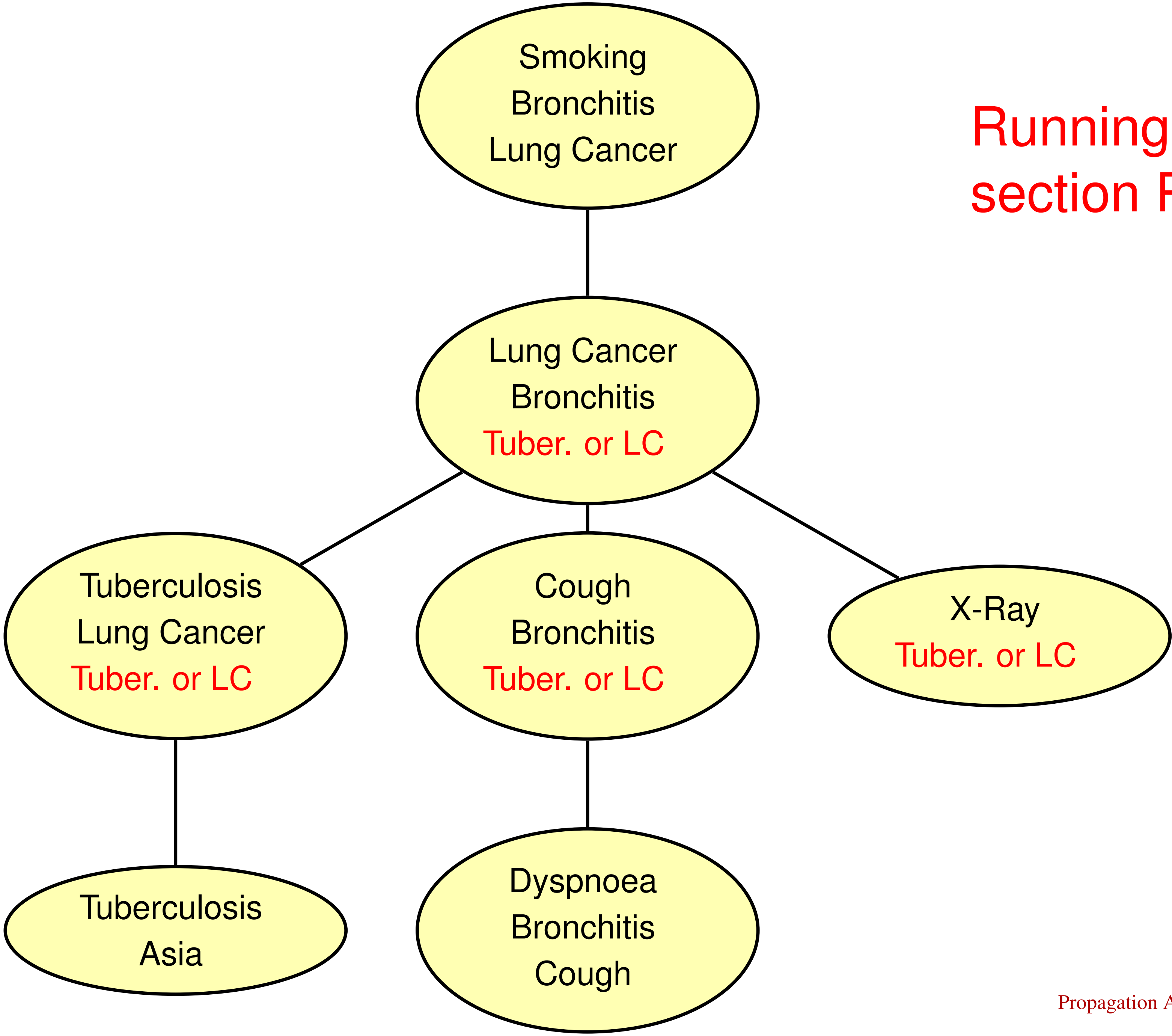


$X - Ray = Yes$

Auxiliar Structure: Joint Tree

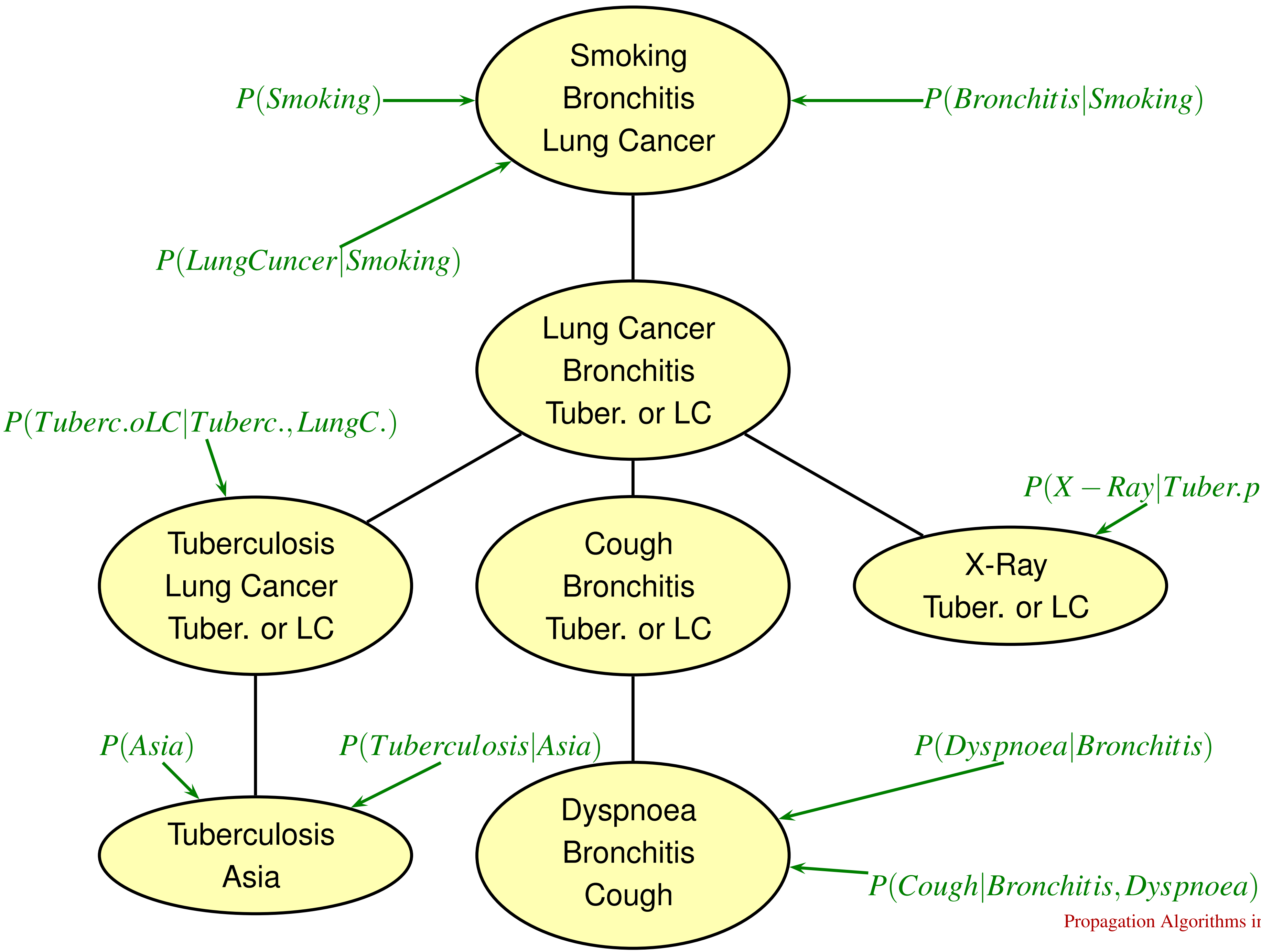


Auxiliar Structure: Joint Tree



Running Intersection Property

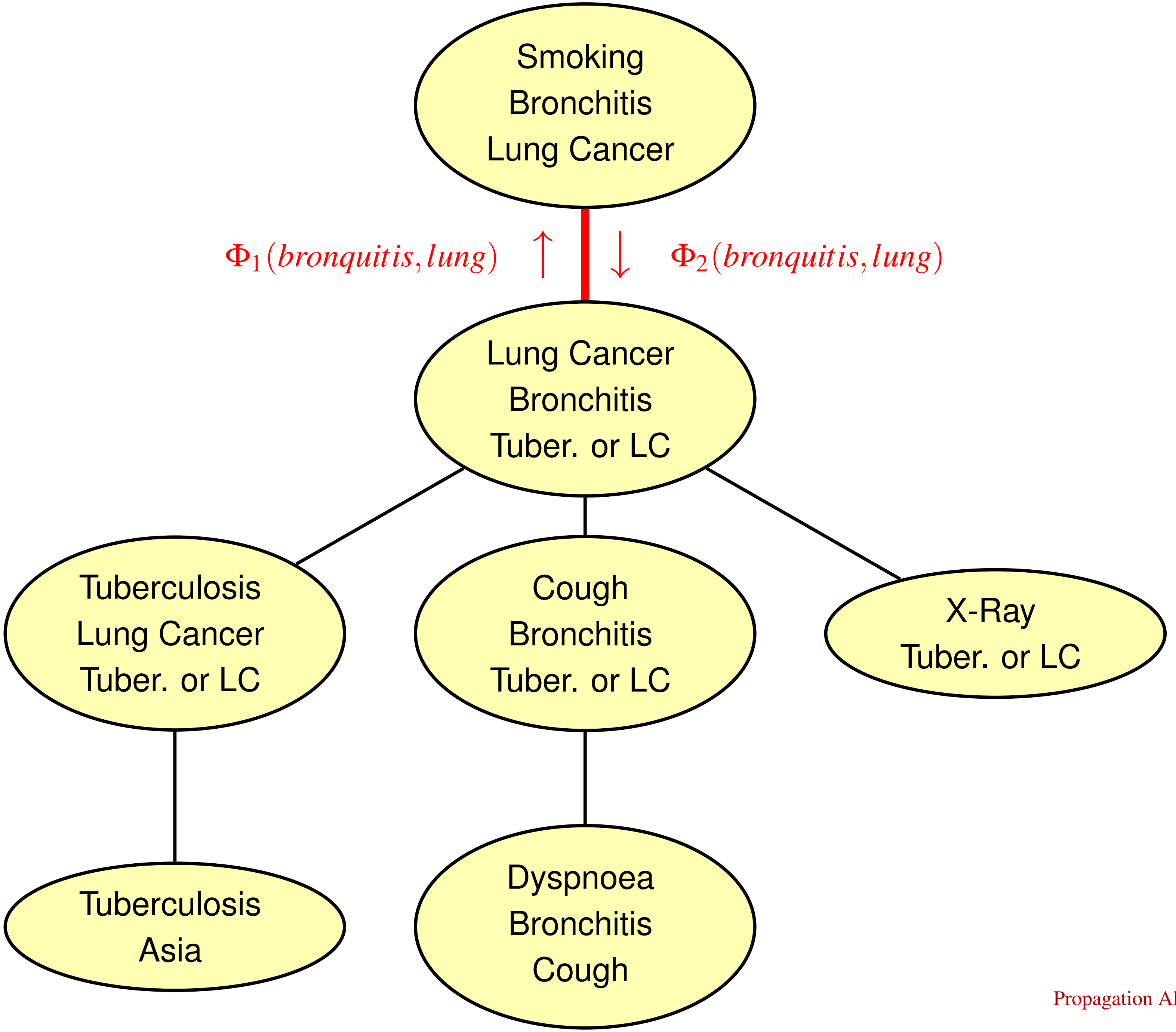
Auxiliar Structure: Joint Tree



Algorithm

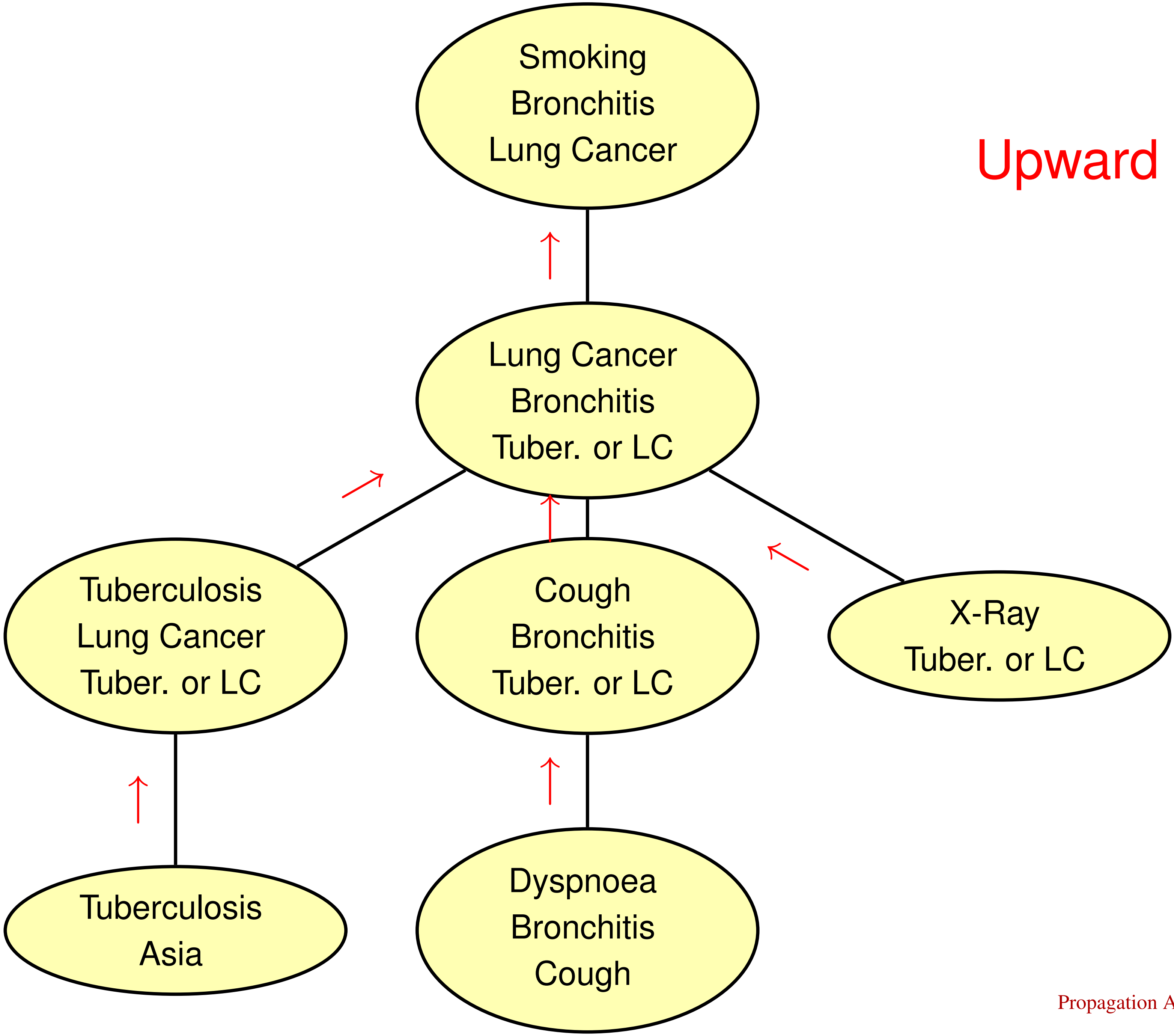
1. Compute the joint tree
2. Assign potentials
3. Incorporate observations (e)
4. Compute messages
5. Compute 'a posteriori' information

Messages



Smoking
Bronchitis
Lung Cancer
$\Phi_1(bronquitis, lung)$ $\uparrow$
$\downarrow$ $\Phi_2(bronquitis, lung)$
Lung Cancer
Bronchitis
Tuber. or LC
Tuberculosis
Lung Cancer
Tuber. or LC
Cough
Bronchitis
Tuber. or LC
X-Ray
Tuber. or LC
Tuberculosis
Asia
Dyspnoea
Bronchitis
Cough

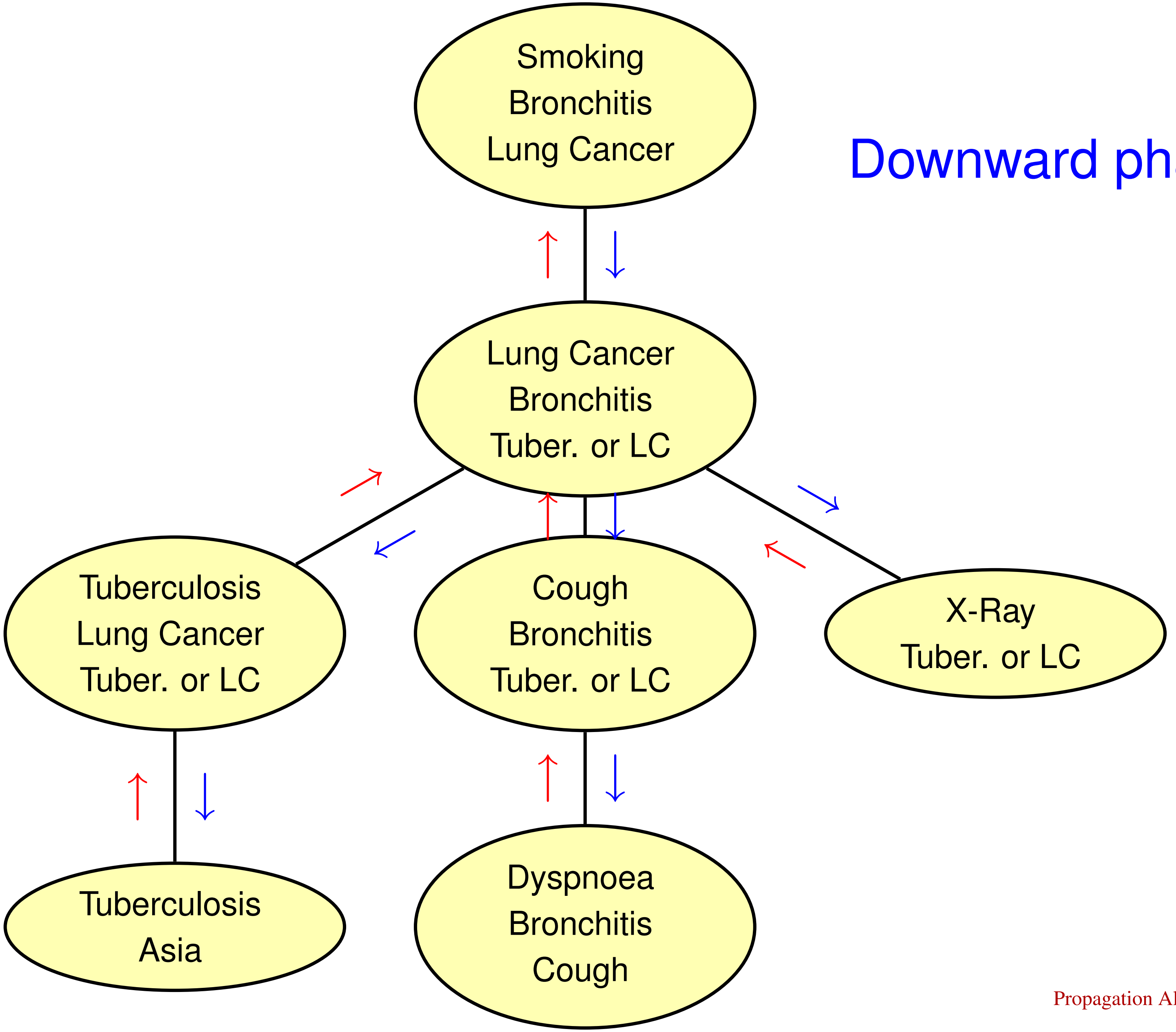
Messages: Computation



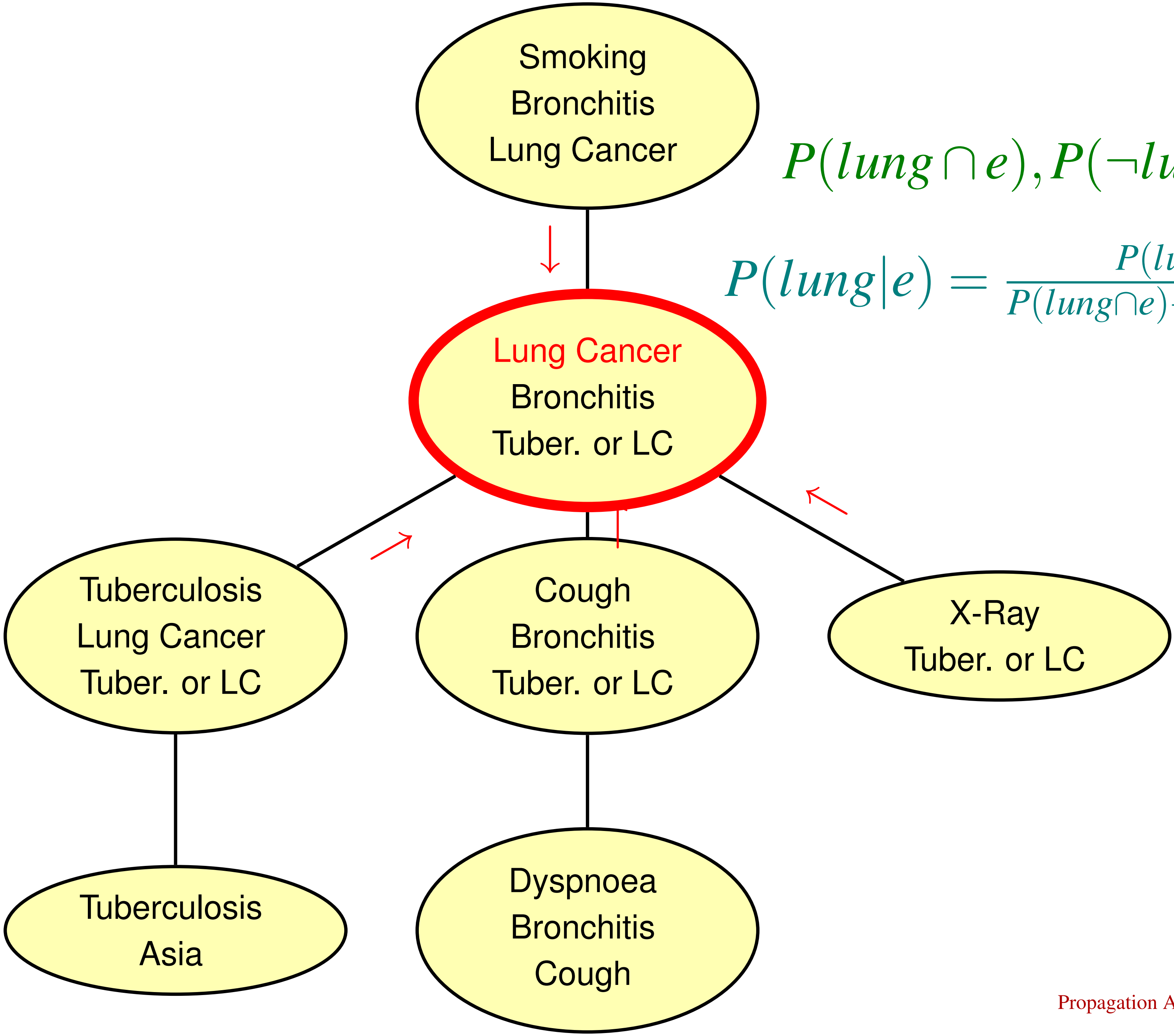
Upward phase

Messages: Computation

Downward phase



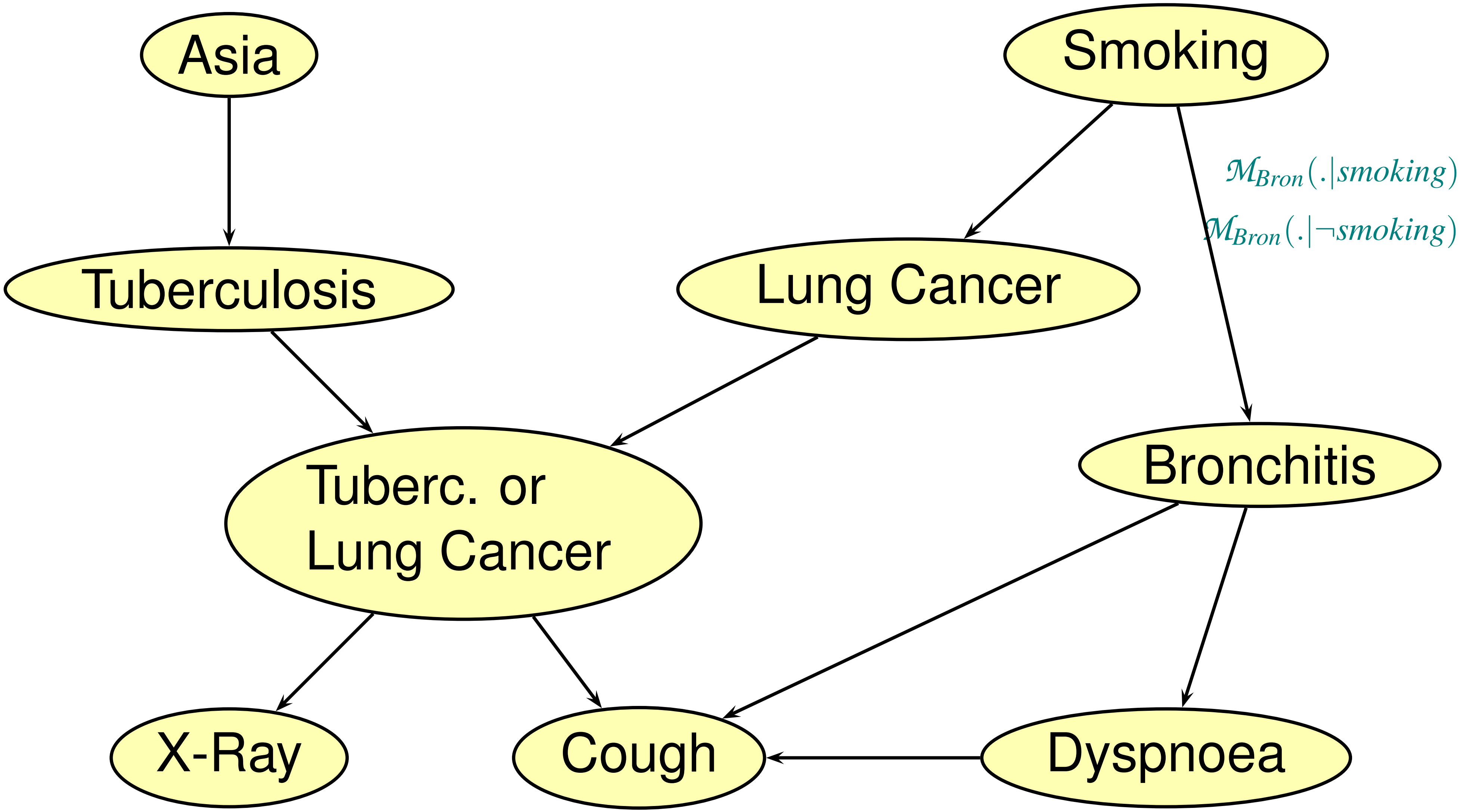
Computing Conditional Probabilities



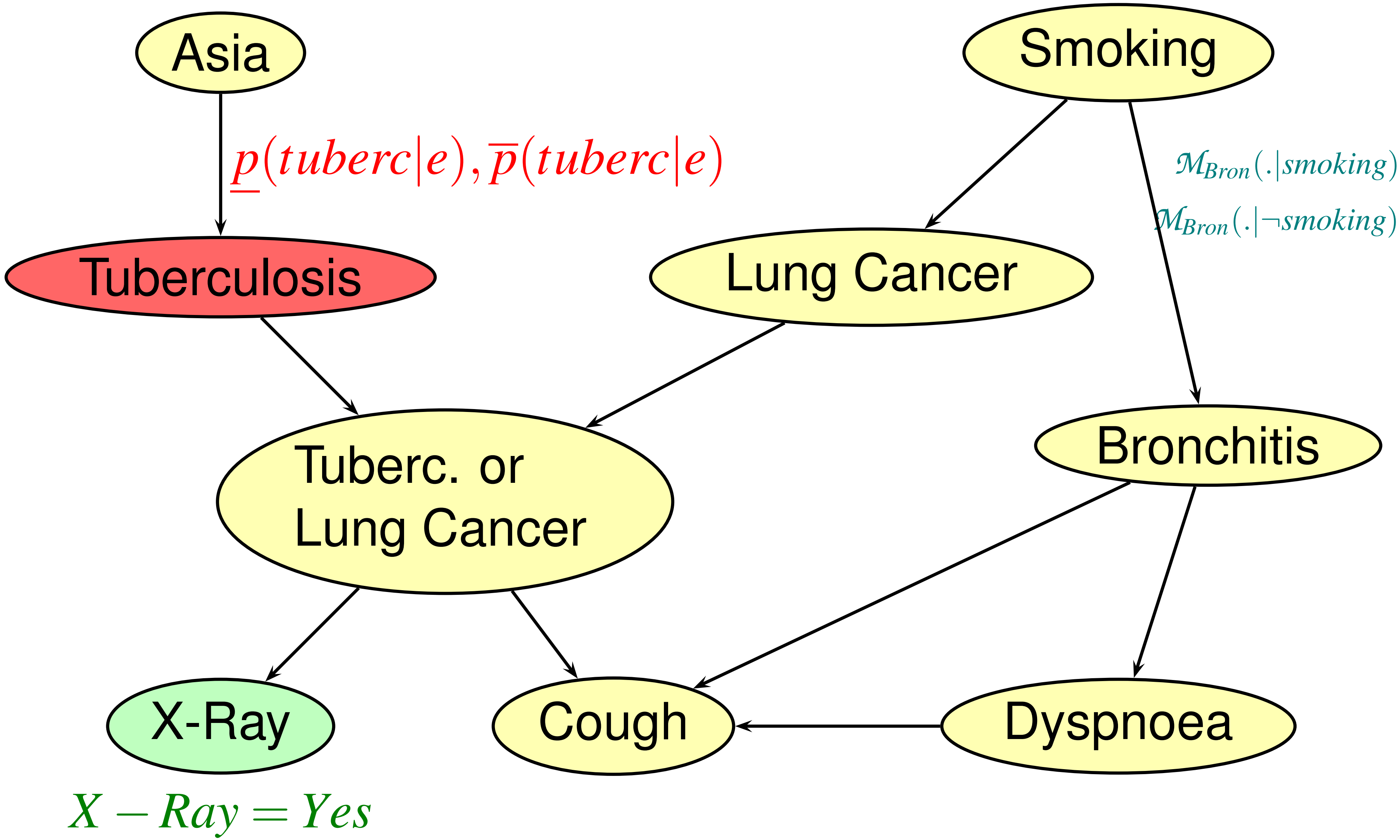
$$P(lung \cap e), P(\neg lung \cap e)$$

$$P(lung|e) = \frac{P(lung \cap e)}{P(lung \cap e) + P(\neg lung \cap e)}$$

Strong Independence



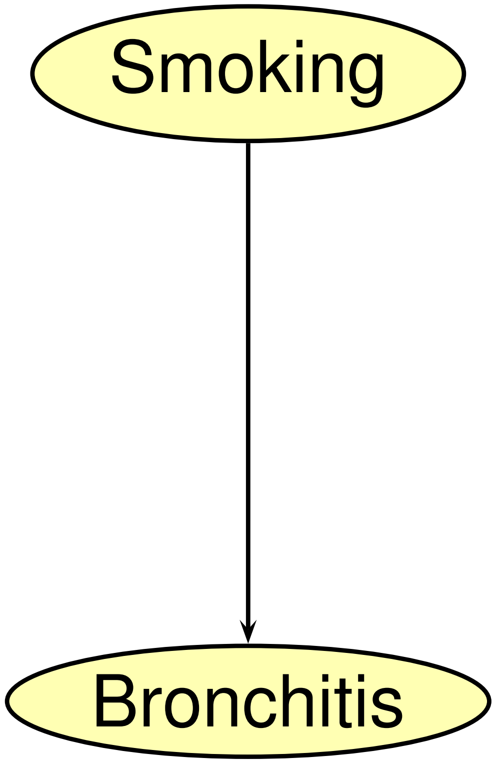
Strong Independence



Computation: Basics

- Usually the credal sets are given by intervals
- The upper and lower probabilities are obtained in the extreme points of the credal sets. So it is enough to consider these extreme points
- It is possible to consider all the possible combinations of the extreme points of conditional credal sets $\mathcal{M}_{Y|X}$, for each one of them compute the conditinal probability of interest and then to obtain the maximum and minimum value
- It is NP-hard to determine the points in which the maximum and minimum is obtained

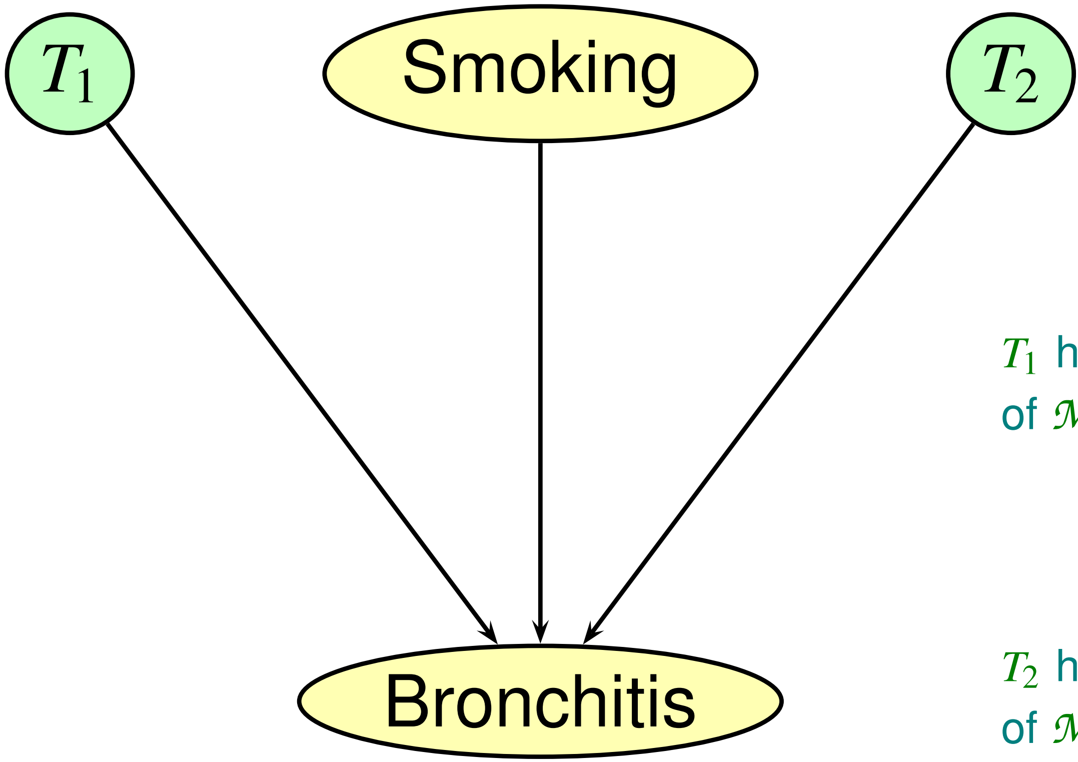
Transparent Nodes



A credal set $\mathcal{M}_{Bron}(.|Smoking)$

A credal set $\mathcal{M}_{Bron}(.|\neg Smoking)$

Transparent Nodes



T_1 has a value for each extreme of $\mathcal{M}_{Bron}(.|Smoking)$

T_2 has a value for each extreme of $\mathcal{M}_{Bron}(.|\neg Smoking)$

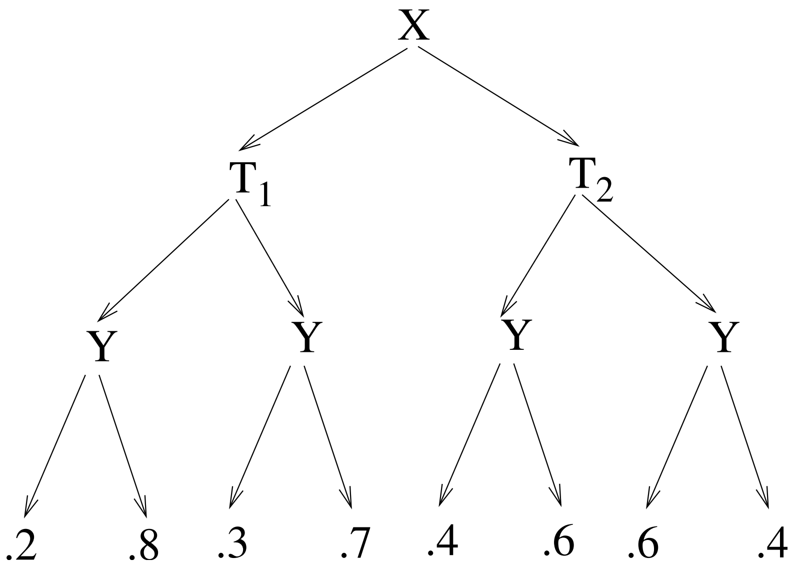
A credal set $\mathcal{M}_{Bron}(.|Smoking)$

A credal set $\mathcal{M}_{Bron}(.|\neg Smoking)$

Example

If the conditional probabilities are given by intervals:

$$P(Y|X) = \left\{ \begin{array}{c|cc} & x_1 & x_2 \\ \hline y_1 & [0.2,0.3] & [0.4,0.6] \\ y_2 & [0.7,0.8] & [0.4,0.6] \end{array} \right.$$



	(x_1,y_1)	(x_1,y_2)	(x_2,y_1)	(x_2,y_2)
P_1	0.2	0.8	0.4	0.6
P_2	0.2	0.8	0.6	0.4
P_3	0.3	0.7	0.4	0.6
P_4	0.3	0.7	0.6	0.4

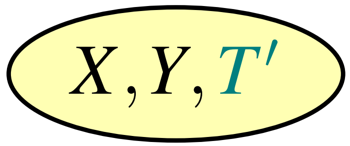
Advantages

- All the information is represented with the same data structure (potentials).
- For each configuration of transparent variables $\mathbf{T} = \mathbf{t}$, we have a joint probability distribution for the rest of variables $(X_1, \ldots, X_n)$, denoted by $P_{\mathbf{t}}$. It can be computed following the same procedure than in the precise probability case.
- We can compute the lower interval for $P(y|e)$ as an optimization problem:

$$\inf_{\mathbf{t}} \; P_{\mathbf{t}}(y|e)$$

- To compute in an exact way we can use modifications of general propagation algorithms, but the general complexity is $\prod_{T \in \mathbf{T}} r_T$ probabilistic propagations, where $\mathbf{T}$ is the set of transparent nodes, and r_T is the number of elements of T.

Transparent Nodes in Joint Trees



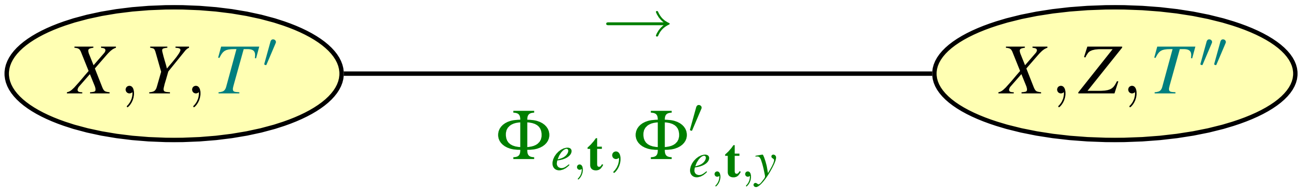
- If we add $\mathbf{T} = \mathbf{t}$ to the evidence e, then after propagation, in any node we can compute $P_{\mathbf{t}}(e)$.
- If we interested in the conditional probability of $Y = y$, then if we add this to the evidence, we can obtain $P_{\mathbf{t}}(e, Y = y)$.
- Dividing we can obtain the desired conditional probability distribution.
- If all the incoming messages to the node containing T' are correct and we want to compute the value of the conditional probability for a configuration equal to the current one, changing $T' = t'_1$ to $T' = t'_2$, we only have to remove one observation in this node and add other one, and then we have the objective function.

A Greedy Algorithm

X, Y, T'

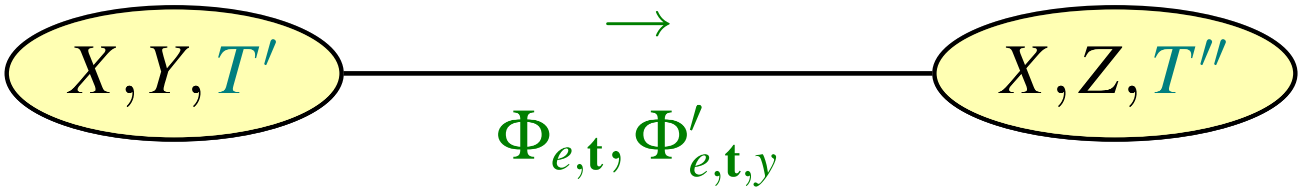
- We start with an arbitrary configuration of transparent variables and carry out a probabilistic propagation.
- We are in a node, compute the objective for the different values of the variable T', and change it to the case with higher value. We add this value to the current evidence.

A Greedy Algorithm



- We start with an arbitrary configuration of transparent variables and carry out a probabilistic propagation.
- We are in a node, compute the objective for the different values of the variable T', and change it to the case with higher value. We add this value to the current evidence.
- We compute two messages to a neighboring group: one with the current evidence (including transparent nodes) and other with the current evidence plus $Y = y$. We repeat optimization for the transparent variables in this new group.

A Greedy Algorithm



- We start with an arbitrary configuration of transparent variables and carry out a probabilistic propagation.
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- We compute two messages to a neighboring group: one with the current evidence (including transparent nodes) and other with the current evidence plus $Y = y$. We repeat optimization for the transparent variables in this new group.
- We stop in a local minimum.

Properties

- It computes inner approximations of the intervals: $[\underline{p}^c(y|e), \overline{p}^c(y|e)] \subseteq [\underline{p}(y|e), \overline{p}(y|e)]$.
- It has been implemented in Elvira system for conditional interval probabilities.
- It is very fast and we have not found a case in which the intervals are not exact yet. The problem is to have examples with imprecision and exact results.

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